REVIEW, MIDTERM #1: MATH 1B

Problem 1. *The integral*

$$\int_a^\infty \frac{1}{x^{\sqrt{3}}}\, dx$$

converges:

(a) *For all values of* $a > 0$*;*
(b) *For all values of* $a \geq 0$*;*
(c) *For* $a = 1$ *only;*
(d) *For no value of* a*.*

Problem 2. *Evaluate the integral*

$$\int \frac{4x^2 + 4x + 1}{4x^2 - 4x + 1}\, dx.$$

Problem 3. *The area of the surface obtained by rotating the curve $y = f(x)$ from $a \le x \le b$ about the x-axis is given by:*

(a) $\int_a^b \pi f(y)^2\, dy$;

(b) $\int_a^b 2\pi y \sqrt{1 + \left(\frac{df}{dx}\right)^2}\, dx$;

(c) $\int_{f(a)}^{f(b)} 2\pi x \sqrt{1 + \left(\frac{df}{dx}\right)^2}\, dy$.

(d) $\int_{f(a)}^{f(b)} 2\pi y \sqrt{1 + \left(\frac{df}{dx}\right)^2}\, dx$.

Problem 4. *Evaluate the integral*

$$\int \sqrt{x} \ln x\, dx.$$

Problem 5. *Which of the following does not have an elementary antiderivative?*

(a) $x^2 \ln x$;
(b) $\sinh x$;
(c) xe^{x^2};
(d) e^{x^2}.

Problem 6. *Evaluate the integral*

$$\int \csc^4 x \, dx.$$

Problem 7. *If the function $f(x)$ is continuous on $-1 \leq x \leq 1$ except at the point $x = a$ for some $-1 < a < 1$, then the improper integral*

$$\int_{-1}^{1} f(x)\,dx$$

is written:

(a) $\lim_{t\to a^+} \int_{-1}^{t} f(x)\,dx + \lim_{t\to a^-} \int_{t}^{1} f(x)\,dx$;
(b) $\lim_{x\to a} \int_{-1}^{x} f(x)\,dx$;
(c) $(df/dx)(a) - \lim_{t\to a} \int_{-t}^{t} f(x)\,dx$;
(d) $\lim_{a\to\infty} \int_{-1}^{1} f(x)\,dx$.

Problem 8. *Evaluate the integral*

$$\int \sqrt{x^2 - 1}\,dx.$$

REVIEW, MIDTERM #1: MATH 1B

Problem 9. *The sequence*

$$a_n = \frac{2n^2}{n^2+1}$$

is:

(a) *Bounded, monotonic, and convergent;*
(b) *Unbounded, monotonic, and not convergent;*
(c) *Bounded, not monotonic, and convergent;*
(d) *Bounded, not monotonic, and not convergent.*

Problem 10. *Find the area of the surface obtained by rotating the curve*

$$y = x^2 + 1$$

about the y-axis for $0 \le x \le 1$.

Problem 11. *The partial fractions decomposition of*

$$\frac{1}{(x-3)^2(x^2+3)}$$

is:

(a) $\dfrac{A}{x-3}+\dfrac{Bx+C}{(x-3)^2}+\dfrac{Dx+E}{x^2+3}$;

(b) $\dfrac{A}{(x-3)^2}+\dfrac{B}{x^2+3}$;

(c) $\dfrac{A}{x-3}+\dfrac{B}{(x-3)^2}+\dfrac{Cx+D}{x^2+3}$;

(d) $\dfrac{Ax+B}{(x-3)^2}+\dfrac{Cx+D}{x^2+3}$;

(e) $\dfrac{A}{x-3}+\dfrac{B}{(x-3)^2}+\dfrac{C}{x^2+3}$.

Problem 12. *Evaluate the integral*

$$\int_0^{\pi} \sin^5 x \cos^2 x \, dx.$$

REVIEW, MIDTERM #1: MATH 1B

Problem 13. *For the integral*

$$\int_0^2 e^x \, dx,$$

given the formula

$$|E_S| \le \frac{K(b-a)^5}{180n^4}$$

with $|f^{(4)}(x)| \le K$ *for* $a \le x \le b$*, the error in approximating the integral using Simpson's rule for* $n = 2$ *is:*

(a) $|E_S| \le 1/90$;
(b) $|E_S| \le e/90$;
(c) $|E_S| \le e^2/90$;
(d) $|E_S| \le e^2/2880$.

Problem 14. *Determine if the integral*

$$\int_1^2 \frac{1}{x\sqrt{\ln x}} \, dx$$

is convergent, and evaluate it if so.

Problem 15. *The sum of the series*

$$1 - \frac{1}{2} + \frac{1}{4} - \frac{1}{8} + \dots$$

is:

(a) 1/2*;*
(b) 1*;*
(c) 2/3*;*
(d) 3/2*;*
(e) *The series is divergent.*

Problem 16. *Evaluate*

$$\int_e^{e^2} \frac{\ln(\ln x)}{x}\, dx.$$

REVIEW, MIDTERM #1: MATH 1B

Problem 17. *For the integral*

$$\int_a^b (2x^2 + 3x - 4)\, dx,$$

which of the following gives the best approximation for fixed n?

(a) *Left endpoint approximation;*
(b) *Right endpoint approximation;*
(c) *Midpoint approximation;*
(d) *The Trapezoidal rule;*
(e) *Simpson's rule.*

Problem 18. *Set up an integral to compute the length of the curve $y = e^x$ for $1 \leq y \leq e$. Suggest a substitution, but do not evaluate completely.*

Problem 19. *If f is continuous, then*

$$\int_{-\infty}^{\infty} f(x)\,dx = \lim_{t\to\infty} \int_{-t}^{t} f(x)\,dx.$$

(a) *True;*
(b) *False;*
(c) *Cannot be determined from the information given.*

Problem 20. *Determine if the integral*

$$\int_{1}^{\infty} \frac{\sqrt{1+\sqrt{x}}}{\sqrt{x}}\,dx$$

is convergent, and evaluate it if so.