REVIEW, MIDTERM #2: MATH 1B

Problem 1. *The equation*

$$e^x y' = x + y$$

is:

(a) *Linear;*
(b) *Separable;*
(c) *Both linear and separable;*
(d) *Neither linear nor separable.*

Problem 2. *Evaluate the limit:*

$$\lim_{x \to 0} \frac{(e^{-x} - 1 + x)^3}{x^2(\cos 2x - 1)^2}.$$

REVIEW, MIDTERM #2: MATH 1B

Problem 3. *Consider the differential equation*

$$y' = -x^2(x^2 + y^2).$$

Which of the following statements is not *true?*

(a) *The solutions to this equation are decreasing functions.*
(b) *Every solution has a critical point.*
(c) *If $y(x)$ is a solution to this equation, then so is $cy(x)$.*
(d) *There are no constant solutions to this equation.*

Problem 4. *Compute a series expansion for*

$$\int \sqrt{1 + x^3}\, dx.$$

Given an estimate for the integral

$$\int_0^1 \sqrt{1 + x^3}\, dx$$

with an error bounded by $1/160$.

REVIEW, MIDTERM #2: MATH 1B

Problem 5. *If* $f(x) = \ln(1 - x^2)$, *what is the value of* $f^{(2n)}(0)$*?*

(a) $1/2n$
(b) $(2n)!/n$
(c) $-1/n$
(d) 0
(e) $-(2n)!/n$
(f) $1/(2n)!$

Problem 6. *Determine if the series*

$$\sum_{n=2}^{\infty} \frac{1}{n \ln n (\ln \ln n)^2}$$

is convergent or divergent.

REVIEW, MIDTERM #2: MATH 1B

Problem 7. *The interval of convergence of the series*

$$f(x) = \sum_{n=1}^{\infty} \frac{(x-6)^n}{3^n(n+2)}$$

is:

(a) $|x-6| < 3$
(b) $|x| < 3$
(c) $[3, 9]$
(d) $3 \leq x < 9$
(e) *The series converges for all* x.

Problem 8. *Solve the initial value problem*

$$(x^2+1)\frac{dy}{dx} = 2xy + 4x^2 + 4, \quad y(0) = 0.$$

REVIEW, MIDTERM #2: MATH 1B

Problem 9. *Which of the following statements is true?*

(a) *If* $\sum_n c_n 2^n$ *converges, then* $\sum_n c_n(-3)^n$ *converges.*
(b) *If* $\sum_n c_n 2^n$ *converges, then* $\sum_n c_n(-2)^n$ *converges.*
(c) *If* $\sum_n c_n(x-3)^n$ *diverges when* $x = 2$*, then it diverges when* $x = 5$*.*
(d) *None of the above statements are true.*

Problem 10. *A tank contains* 100 *L of water with a salt concentration of* 0.01 *kg/L. Brine that contains* 0.02 *kg/L of salt enters the tank at* 5 *L/min. The solution is kept thoroughly mixed and drains from the tank at the same rate. How much salt is in the tank after* t *minutes?*

REVIEW, MIDTERM #2: MATH 1B

Problem 11. *Calculate the Maclaurin series for*

$$f(x) = \frac{x^2}{(1+x)^2}.$$

Problem 12. *Determine if the series*

$$\sum_{n=2}^{\infty} \frac{(-1)^n n}{2n^3 - n - 1}$$

is convergent or divergent.

REVIEW, MIDTERM #2: MATH 1B

Problem 13. *Which of the following statements is true?*

(a) *If* $\sum_n a_n$ *and* $\sum_n b_n$ *are divergent, then so is* $\sum_n (a_n + b_n)$.
(b) *If* $a_n > 0$ *and* $\lim_{n\to\infty}(a_{n+1}/a_n) < 1$, *then* $\lim_{n\to\infty} a_n = 0$.
(c) *If* $a_n > 0$ *and* $\sum_n a_n$ *is divergent, then* $\sum_n \sqrt{a_n}$ *is convergent.*
(d) *If* $a_n, b_n > 0$, *then* $\sum_n (-1)^n(a_n + b_n) = \sum_n (-1)^n a_n + \sum_n (-1)^n b_n$.
(e) *Mathematics is not a rewarding discipline of study.*

Problem 14. *Determine if the series*

$$\sum_{n=1}^{\infty} \frac{2^n}{n\sqrt{n^n}}$$

is convergent or divergent.

REVIEW, MIDTERM #2: MATH 1B

Problem 15. *The series*

$$\sum_{n=1}^{\infty}(-1)^n\frac{\sin n}{n}$$

is:

(a) *Convergent by the alternating series test.*
(b) *Divergent by the test for divergence.*
(c) *Convergent by the integral test.*
(d) *Convergent by the comparison test.*
(e) *None of the above.*

Problem 16. *Find the Taylor series for*

$$f(x) = e^{2x}$$

at $x = 1$. *What is its radius of convergence? Compute the Taylor polynomial* $T_3(x)$ *for* $f(x)$ *at* $x = 1$.

REVIEW, MIDTERM #2: MATH 1B

Problem 17. *Find the orthogonal trajectory of the family of curves given by*

$$y = \frac{k}{1+x^2}.$$

Problem 18. *Determine if the series*

$$\sum_{n=1}^{\infty} \ln\left(\frac{1+1/n}{(n+2)/(n+1)}\right)$$

is convergent or divergent. If it is convergent, find its sum.