REVIEW, FINAL: MATH 1B

Problem 1. *Consider the differential equation*

$$y'' + y = \sin^2 x.$$

Which of the following statements is true?

(a) *Every solution $y(x)$ to this equation is concave up.*
(b) $2\cos x + 3\sin x$ *is a solution.*
(c) *A particular solution can be found using $A\sin^2 x$.*
(d) *If $y_1(x)$ and $y_2(x)$ are solutions to the equation, then so is $y_1(x) + y_2(x)$.*
(e) *None of the above.*

Problem 2. *Use series to compute*

$$\int \frac{e^x - 1}{x}\, dx.$$

What is its radius of convergence?

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Problem 3. *Which of the following functions does* not *have an elementary antiderivative?*

(a) $\dfrac{e^{\tan^{-1} x}}{1+x^2}$
(b) $\sqrt{x^3+1}$
(c) $x^5 e^x$
(d) $\dfrac{\cos(\ln x)}{x}$
(e) $\tan^3 x$.

Problem 4. *Consider the differential equation*

$$y'' + xy' - 2y = 0.$$

Suppose that $y(x) = \sum_{n=0}^{\infty} c_n x^n$ *is a solution to this equation satisfying* $y(0) = 0$, $y'(0) = 1$. *Compute the fourth Taylor polynomial for* $y(x)$ *at* $x = 0$.

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Problem 5. *Consider the antiderivative of a rational function:*

$$F(x) = \int_a^x \frac{P(t)}{Q(t)}\, dt.$$

Which of the following is true?

(a) $F(x)$ *is a rational function.*
(b) $F(x)$ *is a combination of rational functions and logarithms.*
(c) $F(x)$ *is a combination of rational functions, logarithms, and inverse trigonometric functions.*
(d) $F(x)$ *may fail to be an elementary function.*
(e) $\lim_{x\to\infty} F(x) = \infty$.

Problem 6. *Use series to estimate the arc length of* $y = (2/5)x^{5/2}$ *for* $0 \le x \le 1$ *with an error* < 0.02.

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Problem 7. *Let $f(x)$ be a continuous function on the interval $[a, b]$. Which of the following is correct?*

(a) *If $f(x)$ is increasing, then the midpoint approximation gives an upper bound for $\int_a^b f(x)\,dx$.*
(b) *If $f(x)$ is increasing, then the trapezoidal rule gives an upper bound for $\int_a^b f(x)\,dx$.*
(c) *If $f(x)$ is decreasing, then the left endpoint approximation gives an upper bound for $\int_a^b f(x)\,dx$.*
(d) *If $f(x)$ is concave up, then the right endpoint approximation gives a lower bound for $\int_a^b f(x)\,dx$.*
(e) *None of the above.*

Problem 8. *Solve the differential equation*

$$y' = xe^x y(y+1).$$

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Problem 9. *Which of the following is a solution to* $z^4 = -1 + \sqrt{3}i$*?*

(a) $\sqrt[4]{2}e^{5i\pi/6}$
(b) $\sqrt[4]{2}e^{-2i\pi/3}$
(c) $\sqrt[4]{1/8}(-1 + \sqrt{3}i)$
(d) $\sqrt[4]{2}(-\sqrt{3} + i)$.

Problem 10. *Evaluate*

$$\int_{-1}^{1} \frac{1}{\sqrt{4x^2 - x^4}}\, dx.$$

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Problem 11. *For which value of m do we have*

$$\int_0^\pi \cos^2 mx \sin mx \, dx = 2/3?$$

(a) 0
(b) 2
(c) -1
(d) 1
(e) *None of the above.*

Problem 12. *The punch at a party is kept in a* 10 L *bowl and is initially half-full of cranberry juice. Some unruly partygoers decide to spike the punch with PCP and are able to slip a conconction with a toxicity of* 5 mg/L *into the punch at a rate of* 1 L/min. *The punch is being drunk at a rate of* 1/2 L/min. *When the bowl is full, how toxic is the punch?*

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Problem 13. *Which of the following statements is* in*correct?*

(a) *If* $a_n, b_n > 0$, $\sum_n a_n$ *converges and the sequence* $\{b_n/a_n\}$ *converges, then* $\sum_n b_n$ *converges.*
(b) *If* $a_n > 0$ *for all* n *and* $a_n \to 0$, *then* $\sum_n (-1)^n a_n$ *converges.*
(c) *If* $a_n > 0$ *for all* n *and* $\sum_n a_n$ *converges, then* $\sum_n (-1)^n a_n$ *converges.*
(d) *If* $\sum_n a_n$ *converges then* $\sum_n a_n/2^n$ *converges.*
(e) *If* $a_n > 0$ *and* $\sum_n a_n$ *converges then* $\sum_n (-1)^n a_n^2$ *converges absolutely.*

Problem 14. *Determine if the series*

$$\sum_{n=1}^{\infty} (-1)^n (\sqrt[n]{n} - 1)^n$$

is convergent or divergent.