MATH 251: ABSTRACT ALGEBRA I
HOMEWORK #3

Problems (for all)

Problem 1 (sorta DF 1.2.2–3). Let D_{2n} be the dihedral group of order $2n$ with presentation $D_{2n} = \langle r, s \mid r^n = s^2 = 1, rs = sr^{-1} \rangle$. Use these generators and relations to show:

(a) If $x \in D_{2n}$ is a power of r (including $x = r^0 = 1$!), then $rx = xr$ and x has order at most n.

(b) If $x \in D_{2n}$ is not a power of r, then $rx = xr^{-1}$ and x has order 2.

Problem 2 (DF 1.2.10). Let G be the group of rigid motions in $\mathbb{R}^3$ of a cube. Show that G is a nonabelian group of order 24. [Hint: Find the number of positions to which an adjacent pair of vertices can be sent; alternatively, find the number of places to which a given face may be sent and, once a face is fixed, the number of positions to which a vertex on that face may be sent.]

Problem 3 (DF 1.3.1). Let σ be the permutation

$$1 \mapsto 3,\ 2 \mapsto 4,\ 3 \mapsto 5,\ 4 \mapsto 2,\ 5 \mapsto 1$$

and τ be the permutation

$$1 \mapsto 5,\ 2 \mapsto 3,\ 3 \mapsto 2,\ 4 \mapsto 4,\ 5 \mapsto 1.$$

Find the cycle decompositions of each of the following permutations: $\sigma, \tau, \sigma^2, \sigma\tau, \tau\sigma, \tau^2\sigma$.

Problem 4 (DF 1.3.7). Write out the cycle decomposition of each element of order 2 in the symmetric group S_4.

Problem 5. Find the number of elements in the set $\{\sigma \in S_5 : \sigma(2) = 5\}$.

Problem 6. Write out the multiplication tables for D_6 and S_3.

Extra problems (for grad students)

Problem 7. Let G be a finite group.

(a) Prove that, given $a \in G$, there exists a positive integer $n \in \mathbb{Z}_{>0}$, depending on a, such that $a^n = 1$.

(b) Prove that there is an integer $m \in \mathbb{Z}_{>0}$ such that $a^m = 1$ *for all* $a \in G$.

Date: 12 September 2007; due Wednesday, 19 September 2007.