MATH 251: ABSTRACT ALGEBRA I
HOMEWORK #9

Note: This homework is due Wednesday, 14 November 2007.

Problems (for all)

Problem 1. If $\sigma \in \operatorname{Aut}(G)$ and ϕ_g is conjugation by g prove that $\sigma\phi_g\sigma^{-1} = \phi_{\sigma(g)}$. Deduce that $\operatorname{Inn}(G)$ is a normal subgroup of $\operatorname{Aut}(G)$. *[The group* $\operatorname{Out}(G) = \operatorname{Aut}(G)/\operatorname{Inn}(G)$ *is called the* outer automorphism group *of* G*.]*

Problem 2. Determine the isomorphism class of $\operatorname{Inn}(D_8)$. Write out each such inner automorphism explicitly.

Problem 3 (sorta DF 5.3.3). Determine the set of abelian groups of orders 105, 270, and 360, up to isomorphism. For each isomorphism class, give the elementary divisors and invariant factors.

Problems (for grad students)

Problem 4.

(a) Prove that under any automorphism of D_8, r has at most 2 possible images and s has at most 4 possible images. Show that each of these gives rise to an automorphism of D_8, so that $\#\operatorname{Aut}(D_8) = 8$.
(b) Show that $\operatorname{Aut}(D_8) \cong D_8$. *[Hint: Use the fact that a nonabelian group of order* 8 *is isomorphic to* Q_8 *or* D_8*; notice that* $\operatorname{Aut}(D_8)$ *has exactly* 2 *elements of order* 4*.]*

Problem 5 (sorta DF 5.3.9). Let $G = \mathbb{Z}/60\mathbb{Z} \times \mathbb{Z}/45\mathbb{Z} \times \mathbb{Z}/12\mathbb{Z} \times \mathbb{Z}/36\mathbb{Z}$. Find the number of elements in G of order 2 and the number of subgroups of index 2 in G.

Date: 31 October 2007; due Wednesday, 14 November 2007.