MATH 251: ABSTRACT ALGEBRA I
IN CLASS REVIEW, EXAM #1

Problem A.

(a) Let G, H be finite groups with $\#G = \#H > 1$. Give an example of a homomorphism $\phi : G \to H$ that is *not* an isomorphism.

(b) For every positive even integer $n \in \mathbb{Z}_{>0}$, show that there are at least two nonisomorphic groups of order n. Can a group G have $\#G = 0$?

(c) Exhibit elements $a, b \in D_{2n}$ of order 2 such that ab has order n.

Date: 5 October 2007; exam 8 October 2007.

Problem B. Let G, H be groups and $\phi : G \to H$ be a homomorphism.

(a) Prove that the image of ϕ,

$$\phi(G) = \{\phi(g) : g \in G\}$$

is a subgroup of H.

(b) Prove that if ϕ is injective, then $G \cong \phi(G)$.