MATH 251: ABSTRACT ALGEBRA I
REVIEW, EXAM #2

Problem 1. Let G be a group, H a subgroup, and N a normal subgroup. Let $HN = \{hn : h \in H, n \in N\}$. Prove that HN is a subgroup of G.

Problem 2.

(a) Let G be a group, and suppose that G has normal subgroups of orders 2 and 5. Show that G contains an element of order 10.

(b) Give an example to show that a group G may have elements of orders 2 and 5 but no element of order 10.

Problem 3. Let $H = \{\sigma \in S_6 : \sigma(4) = 4\}$. Show that H is *not* a normal subgroup in S_6.

Problem 4. Show that $\mathrm{Inn}(G) = \{1\}$ if and only if G is abelian.

Problem 5. Show that $Z(S_n) = \{()\}$ if $n \geq 3$.

Problem 6. Draw the lattice of subgroups for $\mathbb{Z}/24\mathbb{Z}$.

Problem 7.

(a) Let G be an abelian group and let H be a subgroup of G. Prove that G/H is abelian.

(b) Give an example of a non-abelian group G containing a proper normal subgroup N such that G/N is abelian.

Problem 8. Prove that S_4 has no subgroup isomorphic to Q_8.

Problem 9. Find all conjugacy classes and their sizes in A_4.

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