

**MATH 251: ABSTRACT ALGEBRA I
WORKSHEET, DAY #6**

Problem 1. Which of the following are groups? Justify your answer.

- (a) The set \mathbb{Z} with the operation $*$ defined by $a * b = a - b$;
- (b) The set $\mathbb{Z}/n\mathbb{Z}$ of residue classes modulo n , under the binary operation of multiplication;
- (c) The set of rational numbers (including $0 = 0/1$) whose denominators in lowest terms are odd, under addition;
- (d) The set $\{x \in \mathbb{Q} : |x| < 1\}$, under addition;
- (e) The set $\{x \in \mathbb{Q} : |x| < 1\}$, under multiplication.

Problem 2. Show that any group G with $\#G \leq 4$ is abelian.

Problem 3. Let G be a group with identity e and let $a \in G$. Prove that $a^2 = e$ if and only if a has order 1 or 2.