MATH 251: ABSTRACT ALGEBRA I
WORKSHEET, DAY #37

Problem 1. For each R and I, decide if R is a ring and if I is an ideal of R. If so, describe the quotient ring R/I and the quotient map $\phi : R \to R/I$.

(a) The set $R = \left\{ \frac{a}{2} : a \in \mathbb{Z} \right\} \subset \mathbb{Q}$ and the set $I = \mathbb{Z} \subset R$.

(b) The set $R \subset \mathbb{Q}$ of rational numbers with odd denominator (in lowest terms), and the set $I \subset R$ of rational numbers with even numerator (and odd denominator).

(c) For F a field, the sets

$$R = \left\{ \begin{pmatrix} a & b \\ 0 & d \end{pmatrix} : a, b, d \in F \right\} \subset M_2(F), \quad I = \begin{pmatrix} 0 & b \\ 0 & 0 \end{pmatrix} \subset R.$$

Problem 2. Let R be a commutative ring which has no ideals other than (0) and R. Must R be a field?

Date: Wednesday, 28 November 2007.