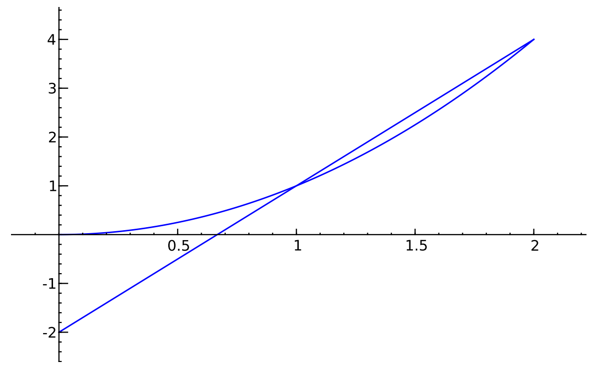
MATH 20C: FUNDAMENTALS OF CALCULUS II
EXAM #2

Problem 1. Find the area of the region between $y = x^2$ and $y = 3x - 2$ from $x = 0$ to $x = 2$. Graph the area of this region.

Solution. We have:



The curves cross when $x^2 = 3x - 2$, or $x^2 - 3x + 2 = (x-2)(x-1) = 0$, so $x = 1$ or $x = 2$. Thus the area is

$$\int_0^1 (x^2 - (3x-2))\,dx + \int_1^2 ((3x-2) - x^2)\,dx = \left(\frac{x^3}{3} - \frac{3x^2}{2} + 2x\right)\Bigg|_0^1 + \left(\frac{3x^2}{2} - 2x - \frac{x^3}{3}\right)\Bigg|_1^2$$
$$= \left(\frac{1}{3} - \frac{3}{2} + 2\right) - 0 + \left(6 - 4 - \frac{8}{3}\right) - \left(\frac{3}{2} - 2 - \frac{1}{3}\right) = 1.$$

Problem 2. Evaluate the integral

$$\int x^{-2} \ln x\,dx.$$

Solution. We use integration by parts, with $u = \ln x$ and $v = x^{-2}$.

	D	I
$+$	$\ln x$	x^{-2}
$-\int$	$1/x$	$-x^{-1}$

So

$$\int x^{-2} \ln x\,dx = -x^{-1}\ln x - \int \frac{1}{x}(-x^{-1})\,dx = -\frac{\ln x}{x} + \int x^{-2}\,dx = -\frac{\ln x}{x} - \frac{1}{x} + C.$$

Problem 3. Evaluate the integral

$$\int_0^\pi 3\cos(2x)\,dx.$$

Solution. We substitute, with $u = 2x$ so that $du = 2\,dx$ or $dx = du/2$. When $x = 0$ we have $u = 0$ and when $x = \pi$ we have $u = 2\pi$. So

$$\int_0^\pi 3x\cos(2x)\,dx = \int_0^{2\pi} \frac{3}{2}\cos(u)\,du = \frac{3}{2}\sin u\Big|_0^{2\pi} = 0 - 0 = 0.$$

Problem 4. The amount of drug in the body of a laboratory rat at time t is given by $D(t) = 3e^{-0.2t}$ where D is in cubic centimeters (cc's) and time t is in hours. What is the average amount of drug in the rat's body over the first 5 hours?

Solution. The average is

$$\frac{1}{5-0}\int_0^5 3e^{-0.2t}\,dt = \frac{3}{5}\frac{e^{-0.2t}}{-0.2}\bigg|_0^5 = -3(e^{-1}-1) = 3(1-1/e) \approx 1.896.$$

Problem 5. Determine if the following given improper integral converges or diverges. If it converges, calculate its value.

$$\int_2^\infty \frac{2}{x^4}\,dx.$$

Solution. We have

$$\int_2^\infty \frac{2}{x^4}\,dx = \lim_{M\to\infty} 2\int_2^M x^{-4}\,dx = \lim_{M\to\infty} 2\frac{x^{-3}}{-3}\bigg|_2^M = \lim_{M\to\infty} -\frac{2}{3x^3}\bigg|_2^M = \lim_{M\to\infty}\left(-\frac{2}{3M^3}+\frac{2}{24}\right) = \frac{1}{12}$$

and the integral converges.

Problem 6. The oil from offshore drilling produces a continuous stream of income of $R(t) = 1000 - 50t$ dollars per year for t years. The revenue is deposited daily into a savings account bearing interest at a rate of 5%. Find the future value of the income stream after the first 20 years of operation.

Solution. We have

$$FV = \int_0^{20}(1000-50t)e^{0.05(20-t)}\,dt = \int_0^{20}(1000-50t)e^{-0.05t+1}\,dt$$

We use integration by parts:

	D	I
$+$	$(1000-50t)$	$e^{-0.05t+1}$
$-$	-50	$\dfrac{e^{-0.05t+1}}{-0.05} = -20e^{-0.05t+1}$
$+\int$	0	$400e^{-0.05t+1}$

Thus

$$\begin{aligned} FV &= \left((1000-50t)(-20e^{-0.05t+1}) + 50(400e^{-0.05t+1})\right)_0^{20} \\ &= 0 + 50(400) - ((1000)(-20e) + 50(400)e) = 20000. \end{aligned}$$