MATH 20C: FUNDAMENTALS OF CALCULUS II
EXAM #3

Name _____

Please complete the following problems in the space provided. You may use an approved calculator. Please include all relevant intermediate calculations and explain your work when appropriate. Be neat and orderly in your answer. Each problem is worth 5 points.

Problem 1.

(a) Which one of the following functions is linear?

$$f(x, y, z) = \frac{2x + 3y - 5z}{7}$$
$$g(x, y, z, w) = x + y + z + w + xy + zw$$
$$h(x, y) = 5x + 6y^2.$$

(b) Your weekly cost (in dollars) to manufacture x gallons of maple syrup and y pounds of maple butter is

$$C(x, y) = 1839 + 30x + 50y.$$

What is the marginal cost of a gallon of maple syrup?

What does the slice $y =$ constant represent?

Problem 2. For the function

$$z = f(x, y) = 2\sqrt{x^2 + y^2} - 9,$$

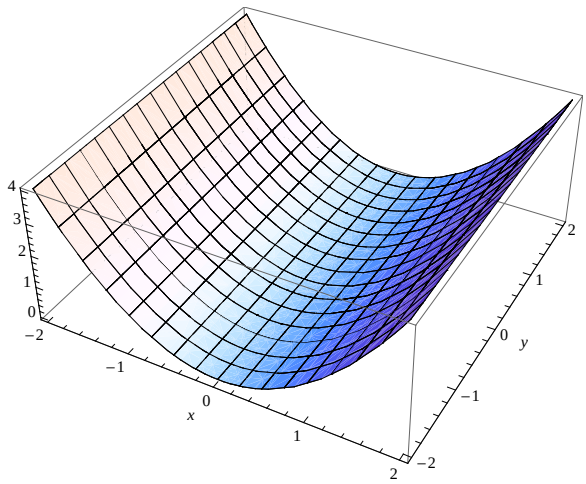
find the equation of the level curve where $z = -5$. Give a description of the graph of this curve.

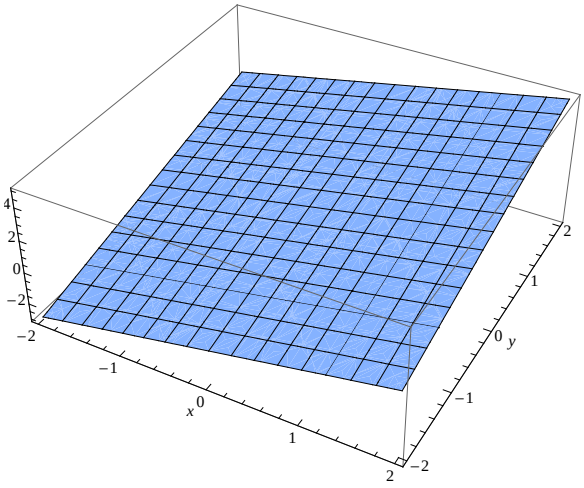
Problem 3. Find the x-, y-, and z-intercepts of the function

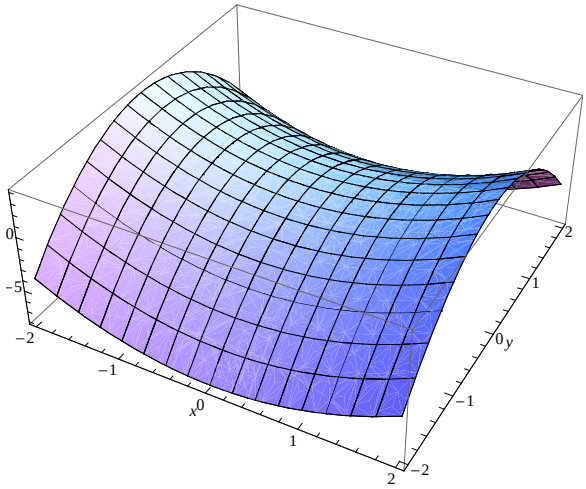
$$z = f(x, y) = y^2 + 2xy + 4x^2 - 4.$$

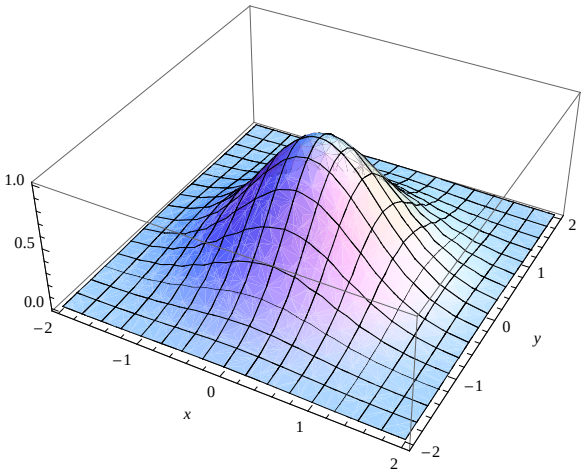
Problem 4. Label each graph below with the corresponding equation.

(a) $f(x, y) = e^{-(x^2+y^2)}$.
(b) $f(x, y) = x^2$.
(c) $f(x, y) = x + y + 1$.
(d) $f(x, y) = x^2 - 2y^2$.









Problem 5. Find the partial derivatives $\dfrac{\partial f}{\partial x}, \dfrac{\partial f}{\partial y}$ of the function

$$f(x, y) = \frac{1}{4x^2 + 3y - 5xy}.$$

Problem 6. Find the partial derivatives $\dfrac{\partial^2 f}{\partial x^2}, \dfrac{\partial^2 f}{\partial x\, \partial y}, \dfrac{\partial^2 f}{\partial y^2}$ for

$$f(x, y) = e^{-2xy}.$$

Problem 7. Locate (but do not classify) all the critical points of the function

$$f(x, y) = xy + \frac{4}{x} + \frac{2}{y}.$$

Problem 8. The function

$$f(x, y) = 2x^2 + y^2 - x^2y^2$$

has a critical point at $(0, 0)$. Determine if this point is a relative maximum, relative minimum, or saddle point.