MATH 20C: FUNDAMENTALS OF CALCULUS II
FINAL EXAM

Name ________________________________

Please circle the answer to each of the following problems. You may use an approved calculator. Each multiple choice problem is worth 2 points.

1. Multiple Choice

Problem 1. If $F(x)$ and $f(x)$ are functions satisfying $F'(x) = f(x)$, then F is called the ____________ of f.

(a) inverse
(b) antiderivative
(c) derivative
(d) composite

Problem 2. When integrating $\int x^3(x^4+1)^2\,dx$ using the substitution method, we would begin by letting u equal:

(a) x^3
(b) $x(x^4+1)$
(c) $(x^4+1)^2$
(d) x^4+1

Problem 3. Suppose you wish to approximate the area bounded by the function $f(x) = 5x^2+2$, the x-axis, and the vertical lines $x=-2$ and $x=4$ using a Riemann sum. If you wish to make eight subintervals, what should be the length of each subinterval?

(a) 0.75
(b) 0.25
(c) 1
(d) Not enough information given.

Problem 4. The area enclosed by the graphs of $y = x^3$ and $y = x$ is given by:

(a) $\displaystyle\int_{-1}^{1}(x^3-x)\,dx$
(b) $\displaystyle\int_{0}^{1}(x^3-x)\,dx$
(c) $\displaystyle\int_{-1}^{0}(x^3-x)\,dx + \int_{0}^{1}(x-x^3)\,dx$
(d) $\displaystyle\int_{-1}^{0}(x-x^3)\,dx + \int_{0}^{1}(x^3-x)\,dx$

Problem 5. The connection between antideriatives and definite integrals is called:

(a) the rule of u-substitution
(b) integration by parts
(c) the fundamental theorem of calculus
(d) the Riemann principle

Problem 6. A function gives the marginal cost of producing ice cream over a ten year period. Interpret the area under the graph of this function.

(a) The total cost of producing ice cream over the ten year period.
(b) The average cost of producing ice cream over the ten year period.
(c) The rate of change in producing ice cream over the ten year period.
(d) None of the above.

Problem 7. Evaluate the definite integral $\int_0^1 x^2 e^x \, dx$.

(a) $e^2 - 2$
(b) $e - 2$
(c) $2e$
(d) $e + 2$

Problem 8. Suppose that the current world population is 5 billion people and the population t years from now is given by the function $P(t) = 5e^{0.023t}$. Determine the average population of the earth during the next 30 years.

(a) 7.2 billion
(b) 6 bilion
(c) 8.8 billion
(d) 5 billion

Problem 9. An integral of the form $\int_0^\infty f(x) \, dx$

(a) cannot have a finite numerical value
(b) may or may not have a finite numerical value
(c) always has a finite numerical value

Problem 10. Evaluate the improper integral $\int_1^\infty e^{-3t} \, dx$.

(a) $3/e^3$
(b) divergent
(c) $1/(3e^3)$
(d) $-1/(3e^3)$

Problem 11. The function $T(m, n) = 3m + 4n + 5mn$ is:

(a) linear
(b) a second-order regression
(c) differential
(d) both linear and a second-order regression

Problem 12. The point $(3, 2, 11)$ will be ____________ units above the $xy-$plane.

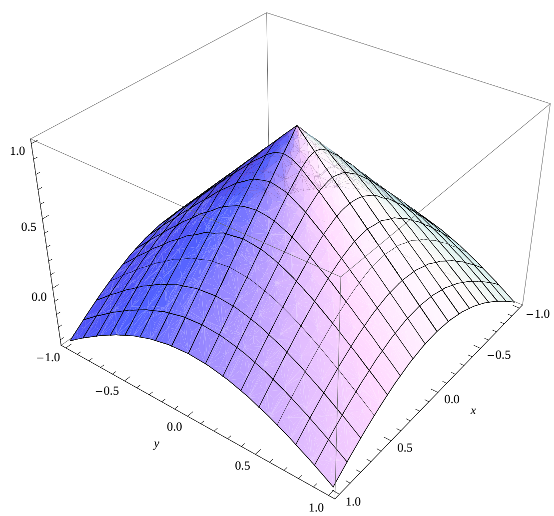
(a) 3
(b) 2
(c) 11
(d) 16

Problem 13. Given the graph of the function $f(x, y)$, if y is set equal to a constant, we will obtain a curve resulting from a slice parallel to the ____________-plane.

(a) xy
(b) xz
(c) yz

Problem 14. Match the graph with one of the equations below:

(a) $f(x, y) = 1 - \sqrt{x^2 + y^2}$
(b) $f(x, y) = (x^2 + y^2)^{-1}$
(c) $f(x, y) = x^2 + y^2$
(d) $f(x, y) = 1 - 3x + 5y$



Problem 15. The graph of the equation $2x + 3y + 3z = 18$ has x-intercept

(a) $(6, 0, 0)$
(b) $(9, 0, 0)$
(c) $(9, 6, 6)$
(d) None of the above

Problem 16. Find f_{yy} if $f(x, y) = x \ln y + ye^x$.

(a) $x + \dfrac{x}{y}$
(b) $-\dfrac{x}{y^2}$
(c) ye^x
(d) $e^x + \dfrac{1}{y}$

Problem 17. If $f(x,y)$ is a function, (a,b) is a critical point, and $H(x,y)$ is the Hessian, then which of the following is true?

(a) $\dfrac{\partial f}{\partial x}(a,b) = \dfrac{\partial f}{\partial y}(a,b) = 0.$
(b) If $f_{xx}(a,b) < 0$ and $H(a,b) > 0$ then (a,b) is a relative minimum.
(c) If $f_{xx}(a,b) > 0$ and $H(a,b) > 0$ then (a,b) is a relative minimum.
(d) Both (a) and (c).

Problem 18. following table shows the number of females residing on US farms in 1990, broken down by age. Numbers are in thousands.

Age	$0-15$	$15-25$	$25-35$	$35-45$	$45-55$	$55-65$	$65-75$	$75-95$
Number	459	265	247	319	291	291	212	126

If X denotes the associated continuous random variable, then $P(15 \leq X \leq 55)$ is:

(a) 0.508
(b) 0.641
(c) 0.376
(d) 0.715

Problem 19. What kind of probability density function is most appropriate for the random variable represented by the time it takes for a cup of coffee to cool to room temperature?

(a) uniform
(b) exponential
(c) normal
(d) none of these

Problem 20. Plutonium-239 decays at a rate of 0.00284% per year. How long in years do you expect it to take for a randomly selected plutonium-239 atom to decay?

(a) 284
(b) $0.0000284e^{0.0000284}$
(c) 24400
(d) 35211

2. Free Response

Please complete the following problems in the space provided. You may use an approved calculator. Please include all relevant intermediate calculations and explain your work when appropriate. Be neat and orderly in your answer. Each free response problem is worth 5 points.

Problem 1. Evaluate the integral $\int \frac{(\ln x)^6}{x}\, dx$.

Problem 2. Calculate the left Riemann sum to approximate $\int_1^3 \frac{1}{1+2x}\, dx$ using $n = 4$ subintervals.

Problem 3. Evaluate the integral $\int (4x^2 - 1)\cos(4x^3 - 3x)\,dx$.

Problem 4. A company produces income at the rate of $f(t) = 1200t$ dollars per year for the next 10 years. Using an annual interest rate of 10%, find the future value of this income stream.

Problem 5. Find the solution to the differential equation $\frac{dy}{dx} = 6xy$ which satisfies $y = -3$ when $x = 0$.

Problem 6. Find $\frac{\partial^2 f}{\partial x^2}, \frac{\partial^2 f}{\partial x \partial y}, \frac{\partial^2 f}{\partial y^2}$ for the function

$$f(x, y) = e^{-xy}.$$

Problem 7. Find the four critical points of the function $f(x, y) = 3x^2y + y^3 - 3x^2 - 3y^2 + 2$.

Problem 8. Show that the function $f(x) = \dfrac{2}{x^2}$ on $[2, \infty)$ is a probability density function.

Problem 9. Use Lagrange multipliers to find the maximum value of the function $f(x, y) = 2xy$ subject to the constrant $x^2 + y^2 = 8$.

Problem 10. A company wishes to design a rectangular box with no top and a volume of 32 cubic inches. Find the dimensions that will minimize the amount of material used.

Thank you for your hard work! It has been a pleasure. Best wishes for the future!