MATH 20C: FUNDAMENTALS OF CALCULUS II
QUIZ #8 (REPEAT)

Problem 1.

(a) Find $\dfrac{\partial f}{\partial x}$ and $\dfrac{\partial f}{\partial y}$ for

$$f(x, y) = (3x^2y^2 - 5x + 6)^4.$$

Solution. We have

$$\frac{\partial f}{\partial x} = 4(6xy^2 - 5)(3x^2y^2 - 5x + 6)^3$$

and

$$\frac{\partial f}{\partial y} = 4(6x^2y)(3x^2y^2 - 5x + 6)^3.$$

(b) Find $\dfrac{\partial^2 f}{\partial x^2}, \dfrac{\partial^2 f}{\partial x\,\partial y}, \dfrac{\partial^2 f}{\partial y^2}$ for

$$f(x, y) = e^{x^2+y^2}.$$

Solution. We have

$$\frac{\partial f}{\partial x} = 2xe^{x^2+y^2}$$
$$\frac{\partial f}{\partial y} = 2ye^{x^2+y^2}$$

so by the product rule (and symmetry)

$$\frac{\partial^2 f}{\partial x^2} = 2e^{x^2+y^2} + 2x(2x)e^{x^2+y^2} = (4x^2 + 2)e^{x^2+y^2}$$
$$\frac{\partial^2 f}{\partial y^2} = (4y^2 + 2)e^{x^2+y^2}$$

and

$$\frac{\partial^2 f}{\partial x\,\partial y} = 2x(2y)e^{x^2+y^2} = 4xye^{x^2+y^2}.$$

Problem 2.

(a) Find all critical points of the function

$$f(x, y) = y^2 - x^2y + 2x^3.$$

Which of the two points is a saddle point?

Solution. We have $f_x = -2xy + 6x^2 = 0$ and $f_y = 2y - x^2 = 0$. Solving for y in the second equation gives $y = x^2/2$, and substituting this into the first equation gives

$$-2x\left(\frac{x^2}{2}\right) + 6x^2 = -x^3 + 6x^2 = -x^2(x - 6) = 0$$

so $x = 0, 6$. Substituting into $y = x^2/2$ gives $y = 0, 18$ so the critical points are $(0, 0)$ and $(6, 18)$.

(b) Compute the Hessian

$$H = f_{xx}f_{yy} - f_{xy}^2.$$

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Solution. We have

$$
\begin{aligned}
f_{xx} &= -2y + 12x \\
f_{xy} &= -2x \\
f_{yy} &= 2
\end{aligned}
$$

so the Hessian is

$$H = (-2y + 12x)2 - (-2x)^2 = -4y + 24x - 4x^2.$$

Then $H(0,0) = 0$ gives us no information about what kind of critical point it is; however, $H(6,18) = -4(18) + 24(6) - 4(6^2) = -72 < 0$ so $(6,18)$ is a saddle point.