MATH 20C: FUNDAMENTALS OF CALCULUS II
QUIZ #8

Problem 1.

(a) Find $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ for

$$f(x,y) = \frac{1}{4xy^2 + 3x + 1}.$$

Solution. We write $f(x,y) = (4xy^2 + 3x + 1)^{-1}$. Then

$$\frac{\partial f}{\partial x} = -(4y^2 + 3)(4xy^2 + 3x + 1)^{-2}$$

and

$$\frac{\partial f}{\partial y} = -(8xy)(4xy^2 + 3x + 1)^{-2}.$$

(b) Find $\frac{\partial^2 f}{\partial x^2}, \frac{\partial^2 f}{\partial x\,\partial y}, \frac{\partial^2 f}{\partial y^2}$ for

$$f(x,y) = e^{xy}.$$

Solution. We have $\frac{\partial f}{\partial x} = ye^{xy}$ and $\frac{\partial f}{\partial y} = xe^{xy}$. So

$$\frac{\partial^2 f}{\partial x^2} = y^2 e^{xy} \qquad \frac{\partial^2 f}{\partial x^2} = x^2 e^{xy}$$

and

$$\frac{\partial^2 f}{\partial x\,\partial y} = \frac{\partial}{\partial x}(xe^{xy}) = e^{xy} + x(ye^{xy}) = (1 + xy)e^{xy}.$$

Problem 2.

(a) Find all critical points of the function

$$f(x,y) = x^2 - xy^2 + \frac{1}{5}y^5.$$

Solution. We set

$$\frac{\partial f}{\partial x} = 2x - y^2 = 0$$

and

$$\frac{\partial f}{\partial y} = -2xy + y^4 = 0.$$

In the first equation, we can solve for x to obtain $2x = y^2$ so $x = y^2/2$; substituting this into the second equation, we get

$$-2(y^2/2)y + y^4 = -y^3 + y^4 = 0$$

or equivalently

$$y^4 - y^3 = y^3(y - 1) = 0$$

so $y = 0, 1$. Substituting these into $x = y^2/2$ gives the critical points $(0,0), (1/2, 1)$.

(b) Compute the Hessian

$$H = f_{xx}f_{yy} - f_{xy}^2.$$

Which of the two critical points is a local minimum?

Date: Wednesday, November 12, 2008.

Solution. We compute that

$$\begin{aligned} f_{xx} &= 2 \\ f_{yy} &= -2x + 4y^3 \\ f_{xy} &= -2y \end{aligned}$$

so

$$H = 2(-2x + 4y^3) - (-2y)^2 = -4x + 8y^3 - 4y^2$$

So $H(0,0) = 0$ and so we cannot determine from the Hessian what kind of critical point it is; however, $H(1/2, 1) = -2 + 8 - 4 = 2 > 0$ and $f_{xx} = 2 > 0$ so the point $(1/2, 1)$ is a local minimum.