# MATH 20C: FUNDAMENTALS OF CALCULUS II
# WORKSHEET, DAY #12

**Problem 1**. Evaluate the integral

$$\int (1-x)e^x\,dx.$$

*Solution.* We use integration by parts. We take $u = 1-x$ and $v = e^x$, since differentiating $u$ makes it simpler. Thus:

| | $D$ | $I$ |
|---|---|---|
| $+$ | $1-x$ | $e^x$ |
| $-\int$ | $-1$ | $e^x$ |

This gives

$$\int (1-x)e^x\,dx = (1-x)e^x - \int (-1)e^x\,dx = (1-x)e^x + e^x + C = (1-x+1)e^x + C = (2-x)e^x + C.$$

**Problem 2**. Evaluate the integral

$$\int (x^2+1)e^{3x+1}\,dx.$$

*Solution.* We choose $u = x^2+1$ and $v = e^{3x+1}$. We have the antiderivative

$$\int e^{3x+1}\,dx = \frac{1}{3}e^{3x+1} + C$$

by the rule

$$\int e^{ax+b}\,dx = \frac{1}{a}e^{ax+b} + C$$

learned in the section on $u$-substitution, or using $u$-substitution directly. This gives:

| | $D$ | $I$ |
|---|---|---|
| $+$ | $x^2+1$ | $e^{3x+1}$ |
| $-$ | $2x$ | $\frac{1}{3}e^{3x+1}$ |
| $+\int$ | $2$ | $\frac{1}{9}e^{3x+1}$ |

So

$$\begin{aligned}\int (x^2+1)e^{3x+1}\,dx &= (x^2+1)\frac{1}{3}e^{3x+1} - 2x\left(\frac{1}{9}e^{3x+1}\right) + \int 2\cdot\frac{1}{9}e^{3x+1}\,dx \\ &= \frac{1}{3}(x^2+1)e^{3x+1} - \frac{2}{9}xe^{3x+1} + \frac{2}{9}\int e^{3x+1}\,dx \\ &= \frac{1}{3}(x^2+1)e^{3x+1} - \frac{2}{9}xe^{3x+1} + \frac{2}{9}\left(\frac{1}{3}e^{3x+1}\right) + C \\ &= \frac{1}{3}(x^2+1)e^{3x+1} - \frac{2}{9}xe^{3x+1} + \frac{2}{27}e^{3x+1} + C.\end{aligned}$$

This answer is correct, but considering that each term has a $e^{3x+1}$ in it, it is worthwhile to simplify this to obtain

$$\begin{aligned}\int (x^2+1)e^{3x+1}\,dx &= \left(\frac{1}{3}x^2 + \frac{1}{3} - \frac{2}{9}x + \frac{2}{27}\right)e^{3x+1} + C \\ &= \left(\frac{1}{3}x^2 - \frac{2}{9}x + \frac{11}{27}\right)e^{3x+1} + C.\end{aligned}$$

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**Problem 3**. Evaluate the integral
$$\int 5x(x-1)^4\,dx.$$

*Solution.* We have:
$$\begin{array}{c|cc} & D & I \\ \hline + & 5x & (x-1)^4 \\ -\int & 5 & (x-1)^5/5 \end{array}.$$
For the antiderivative $\int (x-1)^4\,dx$, we make the substitution $u = x-1$ to get $du = dx$ so
$$\int (x-1)^4\,dx = \int u^4\,du = \frac{u^5}{5} + C = \frac{(x-1)^5}{5} + C.$$
Thus integration by parts gives
$$\begin{aligned}\int 5x(x-1)^4\,dx &= 5x\frac{(x-1)^5}{5} - \int 5\cdot\frac{(x-1)^5}{5}\,dx = x(x-1)^5 - \int (x-1)^5\,dx \\ &= x(x-1)^5 - \frac{(x-1)^6}{6} + C\end{aligned}$$
where the latter follows again by $u$-substitution. In fact, we have the rule
$$\int (ax+b)^n\,dx = \frac{(ax+b)^{n+1}}{a(n+1)} + C$$
whenever $n \neq 1$.

**Problem 4**. Evaluate the integral
$$\int t^{-1/2}\ln t\,dt.$$

*Solution.* Here:
$$\begin{array}{c|cc} & D & I \\ \hline + & \ln t & t^{-1/2} \\ -\int & 1/t & t^{1/2}/(1/2) = 2t^{1/2} \end{array}.$$
So
$$\begin{aligned}\int t^{-1/2}\ln t\,dt &= \ln t\cdot(2t^{1/2}) - \int \frac{1}{t}(2t^{1/2})\,dt = 2t^{1/2}\ln t - 2\int t^{-1/2}\,dt \\ &= 2t^{1/2}\ln t - 2\frac{t^{1/2}}{1/2} + C = 2t^{1/2}\ln t - 4t^{1/2} + C.\end{aligned}$$

**Problem 5**. Evaluate the definite integral
$$\int_1^2 x^2\ln(3x)\,dx.$$

*Solution.* First, we find an antiderivative, then we'll apply the Fundamental Theorem of Calculus. So let's first compute $\int x^2\ln(3x)\,dx$:
$$\begin{array}{c|cc} & D & I \\ \hline + & \ln 3x & x^2 \\ -\int & 1/(3x)\cdot 3 = 1/x & x^3/3 \end{array}.$$
Note that in computing the derivative $(\ln 3x)'$, we had to use the chain rule! So
$$\int x^2\ln(3x)\,dx = \ln 3x\cdot\frac{x^3}{3} - \int \frac{1}{x}\cdot\frac{x^3}{3}\,dx = \frac{1}{3}x^3\ln 3x - \frac{1}{3}\int x^2\,dx = \frac{1}{3}x^3\ln 3x - \frac{x^3}{9} + C.$$
Now evaluating the definite integral, we obtain:
$$\int_1^2 x^2\ln(3x)\,dx = \left(\frac{1}{3}x^3\ln 3x - \frac{x^3}{9}\right)\Bigg|_1^2 = \frac{8}{3}\ln 6 - \frac{8}{9} - \left(\frac{1}{3}\ln 3 - \frac{1}{9}\right) = \frac{8}{3}\ln 6 - \frac{1}{3}\ln 3 - \frac{7}{9}$$
which is a pretty cracked out answer, but hey.