MATH 20C: FUNDAMENTALS OF CALCULUS II
WORKSHEET, DAY #14

Problem 1. Evaluate the integral

$$\int x\cos(-3x^2+4)\,dx.$$

Solution. We make the substitution $u = -3x^2 + 4$, so that $du = -6x\,dx$ or $x\,dx = -du/6$. Then

$$\int x\cos(-3x^2+4)\,dx = \int \cos u \left(\frac{-du}{6}\right) = -\frac{1}{6}\int \cos u\,du = -\frac{1}{6}\sin u + C = -\frac{1}{6}\sin(-3x^2+4) + C.$$

Problem 2. Evaluate the integral

$$\int \left(3\csc(3x) + \frac{1}{x} + 5e^{-x}\right)\,dx.$$

Solution. We have

$$\int \left(3\csc(3x) + \frac{1}{x} + 5e^{-x}\right)\,dx = \int 3\csc(3x)\,dx + \int \frac{1}{x}\,dx + \int 5e^{-x}\,dx = \int 3\csc(3x)\,dx + \ln|x| - 5e^{-x}.$$

For the first integral, we let $u = 3x$ so $du = 3\,dx$ hence

$$\int 3\csc(3x)\,dx = \int \csc u\,du = -\ln|\csc u + \cot u| + C = -\ln|\csc(3x) + \cot(3x)| + C.$$

So altogether,

$$\int \left(3\csc(3x) + \frac{1}{x} + 5e^{-x}\right)\,dx = -\ln|\csc(3x) + \cot(3x)| + \ln|x| - 5e^{-x} + C.$$

Problem 3. Evaluate the integral

$$\int_0^{\pi/2} x\cos x\,dx.$$

Solution. Here we use integration by parts: we take $u = x$ and $v = \cos x$. Thus:

	D	I
$+$	x	$\cos x$
$-$	1	$\sin x$
$+\int$	0	$-\cos x$

.

Thus

$$\int x\cos x\,dx = x(\sin x) - (-\cos x) + C = x\sin x + \cos x + C.$$

So by the Fundamental Theorem of Calculus,

$$\int_0^{\pi/2} x\cos x\,dx = (x\sin x + \cos x)\Big|_0^{\pi/2} = \frac{\pi}{2}\sin(\pi/2) + \cos(\pi/2) - \cos 0 = \frac{\pi}{2} - 1.$$

Problem 4. Evaluate the integral

$$\int e^{2x}\cos x\,dx.$$

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Solution. Here we use integration by parts and the little trick of solving for the unknown integral:

$$\begin{array}{c|cc} & D & I \\ \hline + & e^{2x} & \cos x \\ - & 2e^{2x} & \sin x \\ +\int & 4e^{2x} & -\cos x \end{array}.$$

Thus

$$\int e^{2x}\cos x\,dx = e^{2x}(\sin x) - 2e^{2x}(-\cos x) + \int 4e^{2x}(-\cos x)\,dx = e^{2x}(2\cos x + \sin x) - 4\int e^{2x}\cos x\,dx$$

so solving for the integral, we have

$$5\int e^{2x}\cos x\,dx = e^{2x}(2\cos x + \sin x) + C$$

or

$$\int e^{2x}\cos x\,dx = \frac{1}{5}e^{2x}(2\cos x + \sin x) + C.$$