MATH 20C: FUNDAMENTALS OF CALCULUS II
WORKSHEET, DAY #19

Problem 1. Decide if the integral converges, and if so compute its value.

(a) $\displaystyle\int_0^\infty \frac{2x}{x^2+1}\,dx.$

Solution. By definition,

$$\int_0^\infty \frac{2x}{x^2+1}\,dx = \lim_{M\to\infty}\int_0^M \frac{2x}{x^2+1}\,dx.$$

We substitute $u = x^2+1$ so $du = 2x\,dx$. When $x = 0$ we have $u = 1$ and when $x = M$ we have $u = M^2+1$. Hence

$$\begin{aligned}\lim_{M\to\infty}\int_0^M \frac{2x}{x^2+1}\,dx &= \lim_{M\to\infty}\int_1^{M^2+1}\frac{du}{u} = \lim_{M\to\infty} \ln u|_1^{M^2+1} \\ &= \lim_{M\to\infty} \ln(M^2+1) - \ln 1 = \lim_{M\to\infty} \infty - 0 = \infty\end{aligned}$$

so the integral diverges.

(b) $\displaystyle\int_{-\infty}^{-1} \frac{1}{x^{4/3}}\,dx.$

Solution. We have

$$\begin{aligned}\int_{-\infty}^{-1}\frac{1}{x^{4/3}}\,dx &= \lim_{M\to-\infty}\int_M^{-1} x^{-4/3}\,dx = \lim_{M\to-\infty}\frac{x^{-1/3}}{-1/3}\,dx \\ &= \lim_{M\to-\infty} -\frac{3}{x^{1/3}}\bigg|_M^{-1} = \lim_{M\to-\infty}\left(-\frac{3}{(-1)^{1/3}}+\frac{3}{M^{1/3}}\right) \\ &= 3+0 = 3\end{aligned}$$

and the integral converges.

(c) $\displaystyle\int_0^2 \frac{1}{x^2}\,dx.$

Solution. Here, there is a discontinuity at $x = 0$, since $1/x^2 \to +\infty$ as $x \to 0$. So

$$\int_0^2 \frac{1}{x^2}\,dx = \lim_{a\to 0^+}\int_a^2 x^{-2}\,dx = \lim_{a\to 0^+} -x^{-1}|_a^2 = \lim_{a\to 0^+}\left(-\frac{1}{2}+\frac{1}{a}\right) = -\frac{1}{2}+\infty = \infty.$$

Problem 2. Sales of the text *Calculus and You* have been declining continuously at a rate of 5% per year. Assuming that *Calculus and You* currently sells 5000 copies per year and that sales will continue this pattern of decline, calculate the total future sales of the text.

Solution. Let $S(t)$ denote the rate of sales of the text in t years. Then $S(t) = 5000e^{-0.05t}$ since sales are declining at an exponential rate, and therefore the total sales is

$$\begin{aligned}\int_0^\infty S(t)\,dt &= \lim_{M\to\infty}\int_0^M 5000e^{-0.05t}\,dt = \lim_{M\to\infty} 5000\frac{e^{-0.05t}}{-0.05}\bigg|_0^M \\ &= \lim_{M\to\infty} -100000\left(e^{-0.05M}-1\right) = -100000(0-1) = 100000.\end{aligned}$$

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