MATH 241: ANALYSIS IN SEVERAL REAL VARIABLES I
WORKSHEET, DAY #1

Problem 1. If $\frac{a}{b} < \frac{c}{d}$ with $b > 0$ and $d > 0$, show that $\frac{a+c}{b+d}$ lies between $\frac{a}{b}$ and $\frac{c}{d}$.

Solution. Suppose that $a/b < c/d$. Since $bd > 0$, multiplying by bd we obtain $ad < bc$. Therefore

$$a(b + d) = ab + ad < ab + bc = b(a + c).$$

Since $b(b+d) > 0$, dividing we obtain $a/b < (a+c)/(b+d)$, as desired. The other inequality follows similarly: we have

$$d(a + c) = ad + cd < bc + cd = c(b + d)$$

so $(a + c)/(b + d) < c/d$.

Problem 2. Let

$$S = \{x : x = 5n \text{ for some integer } n\}$$

and let

$$T = \{x : x = 10n \text{ for some integer } n\}.$$

Show in detail that $T \subset S$.

Solution. Let $x \in T$. Then $x = 10n$ for some $n \in \mathbb{Z}$. Thus $x = 5(2n)$ and $2n \in \mathbb{Z}$, so $x \in S$. Thus $T \subset S$.

Problem 3. How many functions are there from the set $\{1, 2, 3, \dots, n\}$ to the set $\{\Box, \diamond, \triangle\}$?

Solution. For each x in the domain $\{1, \dots, n\}$, we have 3 choices for its image. Therefore there are $3n$ possible functions, as each of these is distinct.

Problem 4. Let $f : \mathbb{R} \to \mathbb{R}$ be defined by $f(x) = x^2$ and let $g : \mathbb{R} \to \mathbb{R}$ be defined by $g(x) = x + 1$.

(a) Give formulas which define the maps $f \circ g$ and $g \circ f$, distinguishing which is which.
(b) Is map f injective (one-to-one), surjective (onto), or bijective (a one-to-one correspondence)? What about g?

Solution. For (a), we have $(f \circ g)(x) = f(g(x)) = f(x+1) = (x+1)^2$ and $(g \circ f)(x) = g(f(x)) = g(x^2) = x^2+1$.

For (b), the map f is not injective, as $f(1) = f(-1) = 1$, and is not surjective, since there is no $x \in \mathbb{R}$ such that $f(x) = -1$; consequently f is also not bijective. The map g is bijective (so both injective and surjective) because it has an inverse, namely $h(x) = x - 1$, which is to say $h \circ f = f \circ h$ is the identity map on $\mathbb{R}$.

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