# MATH 241: ANALYSIS IN SEVERAL REAL VARIABLES I
# WORKSHEET #2: EXAM #1 REVIEW

**Problem 1**. Mark each as true or false. Briefly justify your answer.

(a) For any $y \in \mathbb{R}$, there exists $n \in \mathbb{N}$ such that $1/n < y$.

(b) If $S$ is a set and $f : \mathbb{N} \to S$ is injective but not surjective, then $S$ is uncountable.

(c) If $a_n \to 0$, then for every $\epsilon > 0$, there exists $N \in \mathbb{R}$ such that $n > N$ implies $a_n < \epsilon$.

(d) Let $A \subseteq \mathbb{R}$ be bounded and nonempty, and let $c \in \mathbb{R}$. Define $cA = \{ca : a \in A\}$. Then $\sup(cA) = c \sup A$.

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**Problem 2**. Suppose $\sum_n a_n$ diverges. Show that $\sum_n na_n$ also diverges.

**Problem 3**. Prove that the sequence $a_n$ defined by $a_1 = 1$ and $a_{n+1} = \sqrt{1 + a_n}$ converges and find its limit.

**Problem 4**. Suppose that $\lim a_n = a$ with $a > 0$. Show that there exists $N \in \mathbb{N}$ such that $a_n > 0$ for all $n \geq N$.