MATH 241: ANALYSIS IN SEVERAL REAL VARIABLES I
WORKSHEET #3 (PROOF WORKSHOP #2)

Problem 1. Let $A \subseteq \mathbb{R}$ be a nonempty open set. Show that $A \cap \mathbb{Q} \neq \emptyset$.

Problem 2. Prove that a connected set E with at least two distinct elements has no isolated points.

Problem 3. Let $f : A \to \mathbb{R}$ be a function. Suppose there exists $\lambda \in \mathbb{R}_{>0}$ such that

$$|f(x) - f(y)| \leq \lambda |x - y|$$

for all $x, y \in A$. Show that f is uniformly continuous.

Problem 4. Let $f, g : A \to \mathbb{R}$ be continuous functions. Define the function $h : A \to \mathbb{R}$ by $h(x) = \max(f(x), g(x))$. Show that h is continuous.

Problem 5. Let $A, B \subset \mathbb{R}$. Show that $\overline{A \cup B} = \overline{A} \cup \overline{B}$.

Date: Monday, 8 November 2010.