PROBLEM: GIVE A COUNTEREXAMPLE TO THE FOLLOWING CLAIM:

IF $A_1 \supseteq A_2 \supseteq A_3 \supseteq \ldots$ ARE ALL SETS CONTAINING AN INFINITE NUMBER OF ELEMENTS, THEN THE INTERSECTION $\bigcap_{n=1}^{\infty} A_n$ IS INFINITE AS WELL.

PROOOF: $\bigcap_{n=1}^{\infty} A_n = \emptyset$ BECAUSE IF WE DEFINE A_1 AS $\{1,2,3,\ldots\}$ IE $\mathbb{N}$ AND $A_2 = \{2,3,4\ldots\}$, $A_3 = \{3,4,5\ldots\}$ ETC THEN WE CAN SEE THAT THERE IS NO $x \in \bigcap_{n=1}^{\infty} A_n$ BECAUSE ALTHOUGH $x \in A_x$ $x \notin A_{x+1}$. (MORE SPECIFICALLY $1 \in A_1$ BUT $1 \notin A_2, A_3, \ldots$ ETC.)

PROBLEM: LET $A, B \subset \mathbb{R}$. SHOW THAT $(A \cap B)^c \supseteq A^c \cup B^c$.

PROOF: WE WILL ASSUME THERE IS AN ELEMENT

$x \in A^c \cup B^c$ SUCH THAT $A^c \cup B^c = \{x \in S : x \in A^c \text{ OR } x \in B^c\}$

WITH $A^c \subset S$, $B^c \subset S$. WE WILL ALSO ASSUME

$x \in (A \cap B)^c$ SUCH THAT $(A \cap B)^c = \{x \in S : x \notin A \text{ OR } x \notin B\}$

$A^c \cup B^c = \{x \in S : x \in A^c \text{ OR } x \in B^c\}$, SO

$\{x \in S : x \notin A \text{ OR } x \notin B\}$, THEREFORE

$\{x \in S : x \notin A \cap B\} \Rightarrow \{x \in S : x \in (A \cap B)^c\}$

(OR/INTERSECTION)

$\therefore (A \cap B)^c \supseteq A^c \cup B^c$.

PROBLEM: USE THE TRIANGLE INEQUALITY TO PROVE $|a-b| \leq |a| + |b|$.

PROOF: $|(a-c)+(c-b)| \leq |A-C| + |C-B|$

THIS EQUALITY HOLDS FOR ALL REAL NUMBERS, INCLUDING 0

LET $C=0$

$|A-B| \leq |A| + |B|$