MATH 052: INTRODUCTION TO PROOFS
EXAM #1

Problem 1(a). False. Valid expressions would be $P \wedge (\sim Q)$ or $P \wedge (\sim Q \vee R)$.

Problem 1(b). The implication is true, because the hypothesis is false.

Problem 1(c). The contrapositive is "If f is not continuous at $c \in \mathbb{R}$, then f is not differentiable at $c \in \mathbb{R}$".

Problem 1(d). There are 4 elements–no repetitions in sets are allowed.

Problem 1(e). The negation of an implication $P \Rightarrow Q$, which is defined to be $\sim(P \wedge (\sim Q))$, is $P \wedge (\sim Q)$. So the negation is: "You earned a passing grade on the final exam and did not receive a passing grade for your final grade."

Problem 2(a). Fun with truth tables!

P	Q	R	$P \Rightarrow (Q \vee R)$	$(\sim P) \vee R$	$(\sim Q) \Rightarrow ((\sim P) \vee R)$
F	F	F	T	T	T
F	F	T	T	T	T
F	T	F	T	T	T
F	T	T	T	T	T
T	F	F	F	F	F
T	F	T	T	T	T
T	T	F	T	F	T
T	T	T	T	T	T

Since the two columns have the same truth table, the sentential forms are logically equivalent.

Problem 2(b). A tautology is a sentential form which is true no matter what the values of the propositions. For example, $P \vee (\sim P)$ is a tautology.

Problem 3(a). This statement is true: we can take $y = 5 - x$.

Problem 3(b). In symbols, "$(\forall x \in \mathbb{R})(\exists y \in \mathbb{R})(x + y + 3 = 8)$".

Problem 3(c). In symbols, the negation is "$(\exists x \in \mathbb{R})(\forall y \in \mathbb{R})(x + y + 3 \neq 8)$".

Problem 3(d). In words, "There exists a real number x such that for all $y \in \mathbb{R}$ we have $x + y + 3 \neq 8$."

Problem 4. Let $n \in \mathbb{Z}$ be odd. Then $n = 2k + 1$ for some $k \in \mathbb{Z}$. Therefore

$$4n^2 + n - 2 = 4(2k+1)^2 + (2k+1) - 2 = 4(4k^2 + 4k + 1) + (2k+1) - 2 = 2(8k^2 + 9k + 1) + 1.$$

Since $8k^2 + 9k + 1 \in \mathbb{Z}$, by definition, $4n^2 + n - 2$ is odd.

Problem 5(a). $A = \{x \in \mathbb{Z} : -2 \leq x \leq 3 \text{ and } x \neq 0\}$.

Problem 5(b). $A \cup B = \{1, 3, 5, 9, 13, 15\}$, $A \cap B = \{9\}$, and $A \setminus B = \{1, 5, 13\}$.

Problem 5(c). Take $A = \{1, \{1\}\}$, $B = \{1\}$, $C = \{1\}$.

Problem 5(d). We have $A = \{-4, -3, -2, .., 4\}$, $B = \emptyset = E$, and $C = \{2, 0, -2\} = D$.

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