MATH 052: INTRODUCTION TO PROOFS
HOMEWORK #17

Problem 2.8.7(c). We have

$$\begin{aligned}(x,y) \in (A \setminus B) \times C &\Leftrightarrow x \in A \setminus B \text{ and } y \in C \\ &\Leftrightarrow (x \in A \text{ and } x \notin B) \text{ and } y \in C.\end{aligned}$$

Let Q be the proposition $x \in B$ and R the proposition $y \in C$. Then we will use the logical equivalence

$$\sim Q \wedge R \equiv (\sim Q \vee \sim R) \wedge R.$$

This can be proven by a truth table (omitted) or more simply by using the distributive law (1.34) on page 30:

$$(\sim Q \vee \sim R) \wedge R \equiv (\sim Q \wedge R) \vee (\sim R \wedge R) \equiv Q \wedge R$$

since $\sim R \wedge R$ is always false. Therefore

$$\begin{aligned}(x,y) \in (A \setminus B) \times C &\Leftrightarrow x \in A \text{ and } x \notin B \text{ and } y \in C \\ &\Leftrightarrow x \in A \text{ and } (x \notin B \text{ or } y \notin C) \text{ and } y \in C \\ &\Leftrightarrow (x \in A \text{ and } y \in C) \text{ and } (x \notin B \text{ or } y \notin C) \\ &\Leftrightarrow (x \in A \text{ and } y \in C) \text{ and } \sim(x \in B \text{ and } y \in C) \\ &\Leftrightarrow (x,y) \in A \times C \text{ and } (x,y) \notin B \times C \\ &\Leftrightarrow (x,y) \in (A \times C) \setminus (B \times C).\end{aligned}$$

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