MATH 052: INTRODUCTION TO PROOFS
REVIEW, FINAL EXAM

Problem 1. Let $A \subseteq S$. Prove that

$$S \setminus (S \setminus A) = A.$$

Problem 2. A function $f : \mathbb{R} \to \mathbb{R}$ is *continuous* at $c \in \mathbb{R}$ if the following condition holds:

For every $\epsilon > 0$, there exists $\delta > 0$ such that $|f(x) - f(c)| < \epsilon$ whenever $|x - c| < \delta$.

(a) Write the condition in abbreviated form, using quantifiers.
(b) Write the negation of this condition in a quantified form, using no negation symbols.
(c) Write out part (b) mostly in words.

Problem 3. Prove by induction that $n! < n^n$ for all integers $n > 1$.

Problem 4. Show that $\#\mathbb{Z} \leq \#[0, 1]$.

Problem 5. Consider the binary operation $a * b = \dfrac{ab}{3}$ on $\mathbb{Q} \setminus \{0\}$. Show that $*$ is associative and commutative. What is the identity element for $*$?

Problem 6. Prove that if $a \mid b$ then $a^2 \mid b^2$.

Problem 7. Let $\sim$ be an equivalence relation on a set S, and let $a, b \in S$. Show that two equivalence classes under $\sim$ are either equal or disjoint, i.e. either $[a] = [b]$ or $[a] \cap [b] = \emptyset$.

See also:

`http://www.emba.uvm.edu/~sands/m52f11/index.html`.

Date: 7 December 2011; exam 12 December 2011.