MATH 052: INTRODUCTION TO PROOFS
REVIEW, EXAM #2

Problem 1. Let $x \geq -1$. Prove by induction that

$$(1+x)^n \geq 1 + nx$$

for all integers $n \geq 1$.

Problem 2. Give an example of a partition of $\mathbb{Z}$ into four subsets.

Problem 3. Consider the relation R defined on $\mathbb{Z}$ by aRb if and only if $|a-b| \leq 2$. Which of the properties reflexive, symmetric, and transitive does the relation R possess? Justify your answers.

Problem 4. Let $A = \mathbb{R}_{>1} = \{x \in \mathbb{R} : x > 1\}$ and let $B = \mathbb{R}_{>0}$. Show that the map

$$\begin{aligned} f : A &\to B \\ x &\mapsto \frac{5}{x^2 - 1} \end{aligned}$$

is a bijection.

Problem 5. Let A, B, C, D be nonempty sets. Suppose that $A \times B \subseteq C \times D$. Show that $A \subseteq C$ and $B \subseteq D$.

Date: 28 October 2011; exam 2 November 2011.