MATH 052: FUNDAMENTALS OF MATHEMATICS
EXAM #1

Name ______________________________

Problem	Score
1	
2	
3	
4	
5	

Total __________

Date: 24 September 2012.

Problem 1.

(a) Consider the statement:

If $2+2=3$, then the moon is made of cheese.

Is this statement true or false? Briefly explain.

(b) How many elements are in the set $S = \{1, 2, 1, \{1, 2\}, \emptyset\}$?

(c) What is the negation of the statement "If you earn a passing grade on the midterm exam, then you will get your allowance"?

(d) Draw a Venn diagram for two sets A, B and shade the set $(A \cup B) \setminus (A \cap B)$.

Problem 2.

(a) Describe the set $A = \{-2, -1, 1, 2, 3\}$ in the form $\{x \in \mathbb{Z} : P(x)\}$ where $P(x)$ is a proposition depending on the variable x.

(b) Let $A = \{1, 3, 12, 35\}$, $B = \{3, 7, 12, 20\}$ and $C = \{x : x \text{ is a prime number}\}$. List the elements of the following sets. Which of the sets D, E, and F are equal?

$$D = A \cap B$$

$$E = (A \cup B) \setminus C$$

$$F = A \cup (B \setminus C)$$

(c) Give examples of three sets A, B, C such that $B \in A$, $B \subseteq C$, and $A \cap C \neq \emptyset$.

Problem 3. Let $S = \{1, 2, 3\}$. Consider the following open sentences over the domain S:

$$P(n) : \frac{(n+4)(n+5)}{2} \text{ is odd.}$$
$$Q(n) : 2^{n-1} > 3.$$

(a) State $P(1)$ in words.

(b) For which $n \in S$ is

$$P(n) \Leftrightarrow Q(n)$$

true?

(c) Simplify $\sim(P(n) \vee Q(n))$ using de Morgan's laws and state the result in words.

Problem 4.

(a) Show that $(P \Rightarrow R) \vee (\sim Q \Rightarrow R)$ and $(P \vee (\sim Q)) \Rightarrow R$ are not logically equivalent.

(b) What is a tautology?

Problem 5.

(a) Determine the power set $\mathcal{P}(A)$ of the set $A = \{0, 2, -6\}$.

(b) Consider the following subsets of $A = \{1, 2, 3, 4, 5, 6\}$:

$S_1 = \{\{1, 3, 6\}, \{2, 4\}, \{5\}\}$ $\qquad$ $S_2 = \{\{1, 2, 3\}, \{4\}, \emptyset, \{5, 6\}\}$

$S_3 = \{\{1, 2\}, \{3, 4, 5\}, \{5, 6\}\}$ $\qquad$ $S_4 = \{\{1, 4\}, \{3, 5\}, \{2\}\}$

Determine which of these sets are partitions of A.

(c) True or false: a partition of a set A is a subset $\mathcal{S} \subseteq \mathcal{P}(A)$ such that:

(a) $X \neq \emptyset$ for all $X \in \mathcal{S}$;

(b) For all $X, Y \in \mathcal{S}$ either $X = Y$ or $X \cap Y = \emptyset$; and

(c) $\bigcup_{X \in \mathcal{S}} X = A$.

(d) For $A = \{-1, 0, 1\}$ and $B = \{u, v\}$, determine $A \times B$.

(e) Let $S = \{1, 2, 3\}$. Evaluate $\bigcup_{x \in S} [x, 2x]$.