MATH 052: FUNDAMENTALS OF MATHEMATICS
REVIEW, EXAM #1

Problem 1. Give the truth table for $Q \wedge (P \vee (\sim Q))$.

Solution.

P	Q	$P \vee \sim Q$	$Q \wedge (P \vee \sim Q)$
F	F	T	F
F	T	F	F
T	F	T	F
T	T	T	T

Problem 2. I got an A+ in every nuclear physics I took. What is the easiest way for this statement to be true?

Solution. It's easy: I've never taken a nuclear physics class! (An implication $P \Rightarrow Q$ is true if the hypothesis P is false.)

Problem 3. List the elements of the sets

$$A = \{n \in \mathbb{N} : n^3 < 100\} \text{ and } B = \{x \in \mathbb{R} : x^2 + 1 = 0\}.$$

Solution. $A = \{1, 2, 3, 4\}$ and $B = \emptyset$.

Problem 4. Let $A = \{\emptyset, \{\emptyset\}, \{\{\emptyset\}\}\}$.

(a) Determine which of the following are elements of A: $\emptyset$, $\{\emptyset\}$, $\{\emptyset, \{\emptyset\}\}$. Which are subsets of A?
(b) How many elements are in A?
(c) Determine the sets $\{\emptyset\} \cap A$ and $\emptyset \cup A$ and $A \setminus \{\emptyset\}$.

Solution. For (a), we have $\emptyset \in A$, $\{\emptyset\} \in A$, but $\{\emptyset, \{\emptyset\}\} \notin A$; also, we have $\emptyset \subseteq A$, $\{\emptyset\} \subseteq A$, and $\{\emptyset, \{\emptyset\}\} \subseteq A$. Tricky!

For (b), there are three elements in A.

For (c), we have $\{\emptyset\} \cap A = \{\emptyset\}$ and $\emptyset \cup A = A$ and $A \setminus \{\emptyset\} = \{\{\emptyset\}, \{\{\emptyset\}\}\}$. Yikes!

Problem 5. Give an example of a partition of $\mathbb{Z}$ into four subsets.

Solution. Many answers are possible. Here's one:

$$\{\{..., -3, -2, -1\}, \{0\}, \{1\}, \{n \in \mathbb{Z} : n \geq 2\}\}.$$

Problem 6. Consider the statement

"If a series converges, then its terms go to zero."

This statement is in the form $P \Rightarrow Q$. Write each of the corresponding statements and determine their truth value.

(a) $Q \Rightarrow P$.
(b) $\sim Q \Rightarrow \sim P$.
(c) $\sim P \Rightarrow \sim Q$.
(d) $P \Leftrightarrow Q$.
(e) $\sim (P \Rightarrow Q)$.

Which of these are logically equivalent to the original statement?

Solution. P is the proposition "The series converges" and Q is the proposition "The terms of the series go to zero". (The original statement is true!)

Date: 21 September 2012; exam 24 September 2012.

For (a), we have the *converse*: "If the terms of a series go to zero, then it converges." This statement is false, if you remember from calculus.

For (b), we have the *contrapositive*: "If the terms of a series do not go to zero, then it does not converge. This is true, since the original statement is true and the contrapositive is always logically equivalent to the original statement.

For (c), we have: "If a series does not converge, then its terms do not go to zero." This statement is false; probably this is too much calculus for you to remember (good to know, but not on the exam!): an example is the harmonic series $\sum 1/n$.

For (d), we have: "A series converges if and only if its terms go to zero". $P \Leftrightarrow Q$ is the same as $(P \Rightarrow Q)$ and $(P \Leftarrow Q)$; we saw in (a) that the latter is false, so this statement is false.

For (e), remember that an implication $P \Rightarrow Q$ is logically equivalent to $\sim(P \wedge \sim Q)$, so its negation is equivalent to $P \wedge \sim Q$: the only thing that can go wrong is that the hypothesis is true and the conclusion is false. So the answer is: "There exists a series which converges but its terms do not go to zero."

Only the contrapositive is logically equivalent.

Problem 7. Is the following sentence true or false? "This sentence is false."

Solution. The statement is neither true nor false. Do you see why?