MATH 052: FUNDAMENTALS OF MATHEMATICS
WORKSHEET, DAY #24

Problem 1. Prove that for every positive integer n, we have

$$\frac{1}{2\cdot 3}+\frac{1}{3\cdot 4}+\cdots+\frac{1}{(n+1)(n+2)}=\frac{n}{2n+4}.$$

Proof. We use induction. First, the ________________. Since

the formula holds for $n=1$.

Now we prove the _________________________. Assume that

____

holds for $k \in \mathbb{Z}_{>0}$; we show that

____

We have

$$\frac{1}{2\cdot 3}+\frac{1}{3\cdot 4}+\cdots+\frac{1}{(k+1)(k+2)}+\frac{1}{(k+2)(k+3)}=$$

$$\underline{\qquad\quad} + \frac{1}{(k+2)(k+3)} =$$

$$=\frac{k+1}{2k+6}.$$

By the principle of mathematical induction, the result holds for all $n>0$. □

Date: 22 October 2012.

Problem 2. Prove that for all $n \geq 1$, we have $2^n > n$.

Proof. We proceed by induction. The base case is $n =$ ___, and the result holds in this case since

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Now the induction step: assume that

____

and we show that

____

Indeed, we have

$$2^{k+1} = 2 \cdot 2^k > \qquad\qquad\qquad\qquad\qquad\qquad \geq k + 1.$$

By the principle of mathematical induction, the result holds for all $n \geq 1$. □

Problem 3(a). Show that $2x^2 \geq (x+1)^2$ for all real numbers $x \geq 3$.

Proof. We use a direct proof. The inequality

$$2x^2 \geq (x+1)^2 = \underline{\hspace{6cm}}$$

is equivalent to the inequality

$$x^2 \geq \underline{\hspace{6cm}}.$$

Factoring the left-hand side, we have

$$\underline{\hspace{12cm}}.$$

Now our hypothesis $x \geq 3$ means that $x - 1 \geq 2$ so □

Problem 3(b). Show by induction that for all $n \geq 5$ we have $2^n > n^2$. [Use 3(a)!]

Proof. □

Problem 4. Show by induction that for all $n \geq 0$ we have

$$3 \mid (4^n - 1).$$

Proof. $\square$