

MATH 1A: INDETERMINACIES

Here is a classification/table of all possible indeterminacies, and how they are related to LH (L'Hôpital's Rule).

Type	Indeterminant	Determinant
Quotient	$0/0, \infty/\infty$ (with + or -) Apply LH directly	$0/\infty = 0, c/\infty = 0,$ $\infty/c = \pm\infty$ (depending on c)
Product	$0 \cdot \infty, 0 \cdot (-\infty)$ Reciprocate <i>before</i> applying LH	$\infty \cdot \infty = \infty,$ $0 \cdot 0 = 0,$ $c \cdot \infty = \pm\infty$
Difference	$\infty - \infty$ Rewrite and apply PST (e.g. common denominator, factoring highest powers, rationalizing, etc.); only then apply LH	$-\infty - \infty = -\infty,$ $\infty - c = \infty$ $-\infty + c = -\infty,$ etc.
Power	$0^0, \infty^0, 1^\infty$ Add e^{\ln} , then bring the inside power in front of the ln, bring the limit upstairs into the power, and solve a new problem altogether with a product or quotient indeterminacy; finally, substitute the answer into the old problem	$0^1 = 0, 1^0 = 1,$ $\infty^1 = \infty, 1^1 = 1,$ $0^{+\infty} = 0, 3^\infty = \infty,$ $3^{-\infty} = 0,$ $(1/3)^\infty = 0,$ $(1/3)^{-\infty} = \infty$

To summarize, there are 3 types of beasts:

- (1) Indeterminacies: one needs to turn them into a quotient indeterminacy and then apply LH.
- (2) Determinacies: apply LLs and your answer is right there, no need for fancy techniques and such. In fact, LH is plain dangerous to apply when LL works!
- (3) Dubious cases: $1/0$ (don't know), need more info about whether it is 0^+ or 0^- ; $\infty/0$ (don't know), need more info about 0^+ or 0^- ; neither LLs nor LH works in dubious cases. One needs to find more detailed information about the involved zero so as to turn the dubious case into a determinacy and then find the answer directly.

LH alone doesn't solve the problems, and in fact, in tricky problems one has to alternate between LLs, LH, and simplification or other PSTs. There is no general rule which goes when, but as a "rule" of thumb: alternate between LL, LH, simplification, LL, LH, simplification. The end should be with LLs.