QUIZ #2: CALCULUS 1A (Stankova)
Wednesday, February 4, 2004
Section 10:00–11:00 (Voight)

Problem 1. *Evaluate the limit, if it exists:*

$$\lim_{h\to 0} \frac{(3+h)^{-1} - 3^{-1}}{h}.$$

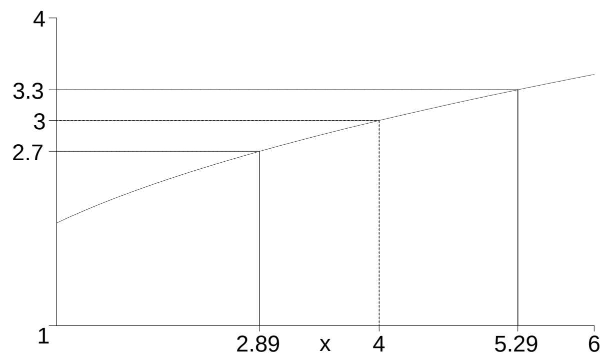
Solution. Applying limit laws, we get the indeterminate expression 0/0. Thus we try to simplify and factor:

$$\begin{aligned}
\lim_{h\to 0} \frac{(3+h)^{-1} - 3^{-1}}{h} &= \lim_{h\to 0} \frac{\dfrac{1}{3+h} - \dfrac{1}{3}}{h} \\
&= \lim_{h\to 0} \frac{1}{h}\left(\frac{1}{3+h} - \frac{1}{3}\right) \\
&= \lim_{h\to 0} \frac{1}{h}\left(\frac{3-(3+h)}{3(3+h)}\right) \\
&= \lim_{h\to 0} \frac{-h}{3h(3+h)} = \lim_{h\to 0} \frac{-1}{3(3+h)} \overset{LL}{=} -1/9.
\end{aligned}$$

QUIZ #2: CALCULUS 1A (Stankova)
Wednesday, February 4, 2004
Section 11:00–12:00 (Voight)

Problem 1. *Use the given graph of f to find a number δ such that*

$$|f(x) - 3| < 0.3 \quad \textit{whenever} \quad 0 < |x - 4| < \delta.$$



Solution. Reading directly from the graph, we see that x can deviate to the left by $4 - 2.89 = 1.11$ and to the right by $5.29 - 4 = 1.29$. We can only deviate to the left or right by the smaller of these values, so we may take $\delta = 1.11$.

Problem 2. *Prove the statement using the ϵ, δ definition of limit. Illustrate with a graph.*

$$\lim_{x \to a} c = c.$$

[Hint: There may be more than one correct answer. Justify your reasoning.]

Solution. We claim that $\lim_{x\to a} c = c$. Let $\epsilon > 0$ be given. Then we need to show that $|f(x) - L| < \epsilon$ whenever $|x - a| < \delta$. But the first inequality is just $|c - c| = 0 < \epsilon$, which is *always* true. Therefore we may choose δ to be any real number.

[If the space aliens came, we would tell them that their inequality is always satisfied, and give them any value of δ no matter what value of ϵ they gave us. Suckers!]