QUIZ #3: CALCULUS 1A (Stankova)
Wednesday, February 11, 2004
Section 10:00–11:00 (Voight)

Problem 1.

(a) *Find the slope of the tangent to the parabola* $y = 1 - 3x + x^2$ *at the point where* $x = a$.

(b) *Find the slope of the tangent lines at the points whose* x*-coordinates are* (i) 0 *and* (ii)1.

Solution. By definition, we have

$$\begin{aligned} m = f'(a) &= \lim_{x \to a} \frac{f(x) - f(a)}{x - a} = \lim_{x \to a} \frac{(1 - 3x + x^2) - (1 - 3a + a^2)}{x - a} \\ &= \lim_{x \to a} \frac{x^2 - a^2 - 3(x - a)}{x - a} = \lim_{x \to a} \frac{(x - a)(x + a - 3)}{x - a} \\ &= \lim_{x \to a} (x + a - 3) = 2a - 3. \end{aligned}$$

[Using the definition

$$f'(a) = \lim_{h \to a} \frac{f(a + h) - f(a)}{h}$$

also is correct and will give the correct answer.]

For part (b), we substitute: $f'(0) = 2(0) - 3 = -3$ and $f'(1) = 2(1) - 3 = -1$.

QUIZ #3: CALCULUS 1A (Stankova)
Wednesday, February 11, 2004
Section 11:00–12:00 (Voight)

Problem 1. *Prove that the equation*

$$x^4 - 3x^2 = 13$$

has at least one real root. Please verify in writing all hypotheses and conditions of any theorem you apply and explain your work.

SOLUTION. We choose $f(x) = x^4 - 3x^2 - 13$, and seek a c such that $f(c) = N = 0$.

Cooking up a and b, we see that $f(0) = -13 < 0$, so we take $a = 0$, and $f(3) = 3^4 - 3^3 - 13 = 41 > 0$, so we take $b = 3$.

The function f is continuous on $[0, 3]$ (actually, on $(-\infty, \infty)$) because it is a polynomial. Therefore, by the Intermediate Value theorem, since

$$f(0) = -13 < 0 < 41 = f(3),$$

there exists a c in $(0, 3)$ such that $f(c) = 0$, i.e. $c^4 - 3c^2 = 13$ as desired.