QUIZ #5: CALCULUS 1A (Stankova)
Wednesday, February 25, 2004
Section 10:00–11:00 (Voight)

Problem 1. *Use logarithmic differentiation to find the derivative of the function*

$$y = x^{\ln x}.$$

SOLUTION. Taking ln of both sides, we obtain

$$\ln y = \ln(x^{\ln x}) = \ln(x)\ln(x) = \ln(x)^2.$$

Differentiating now with respect to x, we have (implicit differentiation)

$$\frac{d}{dx}(\ln y) = \frac{1}{y}\frac{dy}{dx} = \frac{d}{dx}(\ln x)^2.$$

To differentiate the right-side, we apply the chain rule:

$$\frac{d}{dx}(\ln x)^2 = 2\ln x \frac{d}{dx}(\ln x) = \frac{2\ln x}{x}.$$

Therefore we get

$$\frac{1}{y}\frac{dy}{dx} = \frac{2\ln x}{x}$$

so

$$\frac{dy}{dx} = y\frac{2\ln x}{x} = x^{\ln x}\frac{2\ln x}{x}.$$

QUIZ #5: CALCULUS 1A (Stankova)
Wednesday, February 25, 2004
Section 11:00–12:00 (Voight)

Problem 1. *Differentiate the function*

$$f(x) = \sin(\ln(2x)).$$

SOLUTION. We use the chain rule:

$$f'(x) = \cos(\ln(2x))(\ln 2x)' = \cos(\ln(2x))\frac{2}{2x} = \frac{\cos(\ln(2x))}{x}.$$

Problem 2. *Differentiate the function*

$$h(x) = \ln(x + \sqrt{x^2 - 1}).$$

SOLUTION. We find the derivative again using the chain rule

$$h'(x) = (\ln(x + \sqrt{x^2-1}))' = \frac{1}{x + \sqrt{x^2-1}}(x + \sqrt{x^2-1})'.$$

The second part is

$$\begin{aligned}(x + \sqrt{x^2-1})' &= (x + (x^2-1)^{1/2})' = 1 + \frac{1}{2}(x^2-1)^{-1/2}(2x) \\ &= 1 + \frac{x}{\sqrt{x^2-1}} = \frac{x + \sqrt{x^2-1}}{\sqrt{x^2-1}}.\end{aligned}$$

Altogether, this gives the derivative

$$h'(x) = \frac{1}{x + \sqrt{x^2-1}} \cdot \frac{x + \sqrt{x^2-1}}{\sqrt{x^2-1}} = \frac{1}{\sqrt{x^2-1}}.$$