QUIZ #6: CALCULUS 1A (Stankova)

Wednesday, March 3, 2004

Section 10:00–11:00 (Voight)

Problem 1. *Find y' if*

$$y = \ln\left(\frac{2x}{x^2-y^2}\right).$$

SOLUTION. We have

$$\begin{aligned}
y' &= \frac{1}{\dfrac{2x}{x^2-y^2}}\left(\frac{2x}{x^2-y^2}\right)' \\
&= \frac{x^2-y^2}{2x}\,\frac{(x^2-y^2)(2)-2x(2x-2yy')}{(x^2-y^2)^2} \\
&= \frac{2x^2-2y^2-4x^2+4xyy'}{2x(x^2-y^2)} \\
&= \frac{-x^2-y^2+2xyy'}{x(x^2-y^2)} \\
x(x^2-y^2)y' &= -x^2-y^2+2xyy' \\
(x^3-xy^2-2xy)y' &= -x^2-y^2 \\
y' &= \frac{-x^2-y^2}{x^3-xy^2-2xy} = \frac{x^2+y^2}{xy^2+2xy-x^3}.
\end{aligned}$$

Note the cancellation of the (x^2-y^2) terms in the third step!

QUIZ #6: CALCULUS 1A (Stankova)
Wednesday, March 3, 2004
Section 11:00–12:00 (Voight)

Problem 1. *Find an equation of the tangent line to the curve*

$$y = \frac{|\ln x|}{x^2+1}$$

at the point $(2, (\ln 2)/5)$. *Simplify if you want full credit.*

SOLUTION. Since $\ln 2 > 0$, we know that $\ln x > 0$ when x is near $x = 2$, hence $|\ln x| = \ln x$ near $x = 2$. Therefore by the quotient rule, near $x = 2$ we have

$$y' = \frac{(x^2+1)(1/x) - (\ln x)(2x)}{(x^2+1)^2}$$

so

$$y'(2) = \frac{5/2 - 4\ln 2}{25} = \frac{5 - 8\ln 2}{50}.$$

Therefore by the point-slope formula, the tangent line is

$$y - \frac{\ln 2}{5} = \left(\frac{5 - 8\ln 2}{50}\right)(x-2).$$

This is enough simplification, really, but we can also write

$$50y - 10\ln 2 = (5 - 8\ln 2)(x-2) = (5 - 8\ln 2)x - 10 + 16\ln 2$$

so

$$50y = (-8\ln 2 + 5)x + (26\ln 2 - 10).$$