QUIZ #7: CALCULUS 1A (Stankova)
Wednesday, March 10, 2004
Section 10:00–11:00 (Voight)

Problem 1. *Evaluate*

$$\lim_{x\to e} \frac{e^{\ln x} - e}{x - e}.$$

Explain your work.

Solution. We match to the template

$$f'(a) = \lim_{x\to a} \frac{f(x) - f(a)}{x - a}.$$

We take $f(x) = e^{\ln x}$ and $a = e$, and verify that $f(a) = e^{\ln e} = e$ indeed. Therefore the limit is equal to $f'(e)$. We compute

$$f'(x) = \frac{1}{x} e^{\ln x}$$

so $f'(e) = e/e = 1$, which is equal to the limit.

Or, you can simply note that $e^{\ln x} = x$, so we just have

$$\lim_{x\to e} \frac{x - e}{x - e} = 1.$$

Problem 2. *Evaluate*

$$\lim_{x\to 0} \frac{\sin((3 + x)^2) - \sin 9}{x}.$$

Explain your work.

Solution. We again match to the template, and let $f(x) = \sin((3+x)^2)$ and $a = 0$. We verify that $f(0) = \sin 9$, so the limit is equal to $f'(0)$. We compute by the chain rule:

$$f'(x) = \cos((3 + x)^2)((3 + x)^2)' = \cos((3 + x)^2)(2(3 + x))$$

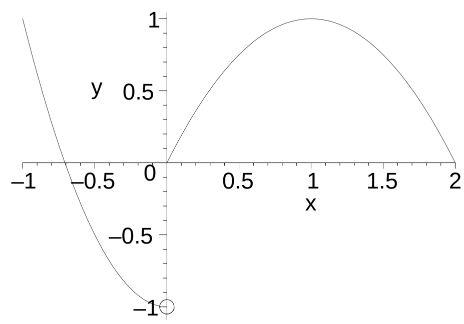
and so $f'(0) = 6\cos 9$, the value of the limit.

QUIZ #7: CALCULUS 1A (Stankova)
Wednesday, March 10, 2004
Section 11:00–12:00 (Voight)

Problem 1. *Sketch the graph of f by hand and use your sketch to find the absolute (global) and local maximum and minimum values of f.*

$$f(x) = \begin{cases} 2x^2 - 1, & \text{if } -1 \leq x < 0; \\ 1 - (x-1)^2, & \text{if } 0 \leq x \leq 2. \end{cases}$$

SOLUTION. We have the graph:



Looking at the graph, we see that f has an absolute maximum at $x = -1$ and $x = 1$. It has no local or absolute minimum, because if it did it would be at $x = 0$ but f jumps at that point. It has a local maximum at $x = 1$.