QUIZ #8: CALCULUS 1A (Stankova)
Wednesday, March 17, 2004
Section 10:00–11:00 (Voight)

Problem 1. *Verify that the function*

$$f(x) = x^3 + 2x - 2$$

satisfies the hypotheses of the Mean Value Theorem on the interval $[0, 1]$.

Then find all numbers c that satisfy the conclusion of the Mean Value Theorem.

SOLUTION. The function f is continuous on $[0, 1]$ and differentiable on $(0, 1)$ because it is a polynomial, therefore it satisfies the MVT.

We compute that

$$f'(x) = 3x^2 + 2$$

and

$$\frac{f(1) - f(0)}{1 - 0} = 1 - (-2) = 3$$

so we solve the equation $3x^2+2 = 3$, or $3x^2 = 1$ or $x^2 = 1/3$, i.e. $x = \pm 1/\sqrt{3}$. We care only for values c in the interval $(0, 1)$, so we have only $c = 1/\sqrt{3}$.

Problem 2. *Let* $f(x) = x^2/(x - 2)$. *Show that there is no value of c such that*

$$f(3) - f(1) = f'(c)(3 - 1).$$

Does this contradict the Mean Value Theorem? Why or why not?

SOLUTION. We have $f(3) - f(1) = 9 - (-1) = 10$, so we want to show there is no c such that $f'(c) = 5$. Well,

$$f'(x) = \frac{(x - 2)(2x) - x^2}{(x - 2)^2} = \frac{x^2 - 4x}{x^2 - 4x + 4} = 5$$

which becomes

$$\begin{aligned} x^2 - 4x &= 5(x^2 - 4x + 4) = 5x^2 - 20x + 20 \\ 0 &= 4x^2 - 16x + 20 = 4(x^2 - 4x + 5). \end{aligned}$$

Applying the quadratic formula, we get

$$x = \frac{4 \pm \sqrt{16 - 20}}{2}$$

therefore the original quadratic does not have any real roots. Therefore no such c exists.

The does not contradict the Mean Value Theorem because the original function is discontinuous at $x = 2$.

QUIZ #8: CALCULUS 1A (Stankova)
Wednesday, March 17, 2004
Section 11:00–12:00 (Voight)

Problem 1. *Let $f(x) = xe^x$.*

(a) *On what intervals is f increasing or decreasing? (Open or closed intervals are acceptable.) Explain your work.*
(b) *Find the local maximum and minimum values of f. Explain.*
(c) *Find the intervals of concavity and the inflection points of f.*
(d) *Draw the graph of f.*

SOLUTION. For (a), we compute that

$$f'(x) = e^x + xe^x = (1 + x)e^x.$$

Note that $e^x > 0$ for all x. Therefore $f'(x) > 0$ and f is increasing for $1+x > 0$, i.e. $x > -1$, and similarly f is decreasing for $x < -1$. Therefore x is decreasing on the interval $(-\infty, -1]$ and increasing on the interval $[-1, \infty)$.

For (b), we see that $f'(x) = (1+x)e^x = 0$ so $1+x = 0$, therefore $x = -1$ is the only critical point. We compute

$$f''(x) = (1 + x)e^x + e^x = (2 + x)e^x$$

and $f''(-1) = e^{-1} > 0$ so $x = -1$ is a local minimum. There is no local maximum.

For (c), we see again since $e^x > 0$ that $f''(x) > 0$ for $2 + x > 0$, or $x > -2$. Therefore f is concave upward on $(-2, \infty)$ and concave downward on $(-\infty, -2)$. So $x = -2$ is an inflection point.

Finally we have the graph for (d):