QUIZ #9: CALCULUS 1A (Stankova)
Wednesday, March 31, 2004
Section 10:00–11:00 (Voight)

Problem 1. *Find the limit. Use l'Hôpital's Rule where appropriate.*

$$\lim_{x\to\pi/2^+} \frac{\ln(x-\pi/2)}{\tan x}$$

SOLUTION. We have:

$$\lim_{x\to\pi/2^+} \frac{\ln(x-\pi/2)}{\tan x} = \frac{-\infty}{-\infty} = \lim_{x\to\pi/2^+} \frac{1/(x-\pi/2)}{\sec^2 x} = \lim_{x\to\pi/2^+} \frac{\cos^2 x}{x-\pi/2} = \frac{0}{0}$$
$$= \lim_{x\to\pi/2^+} \frac{2\cos x(-\sin x)}{1} = 0.$$

Problem 2. *Find the limit. Use l'Hôpital's Rule where appropriate.*

$$\lim_{x\to 0^+} \sin(2x)\ln x.$$

SOLUTION. We have:

$$\lim_{x\to 0^+} \sin(2x)\ln x = 0(-\infty) = \lim_{x\to 0^+} \frac{\ln x}{(\sin 2x)^{-1}}$$
$$= \lim_{x\to 0^+} \frac{1/x}{-(\sin 2x)^{-2}(-2\cos 2x)} = \lim_{x\to 0^+} \frac{(\sin 2x)^2}{2x\cos 2x} = \frac{0}{0}$$
$$= \lim_{x\to 0^+} \frac{2(\sin 2x)(2\cos 2x)}{2\cos 2x + 2x(-2\sin 2x)} = \frac{0}{2+0} = 0.$$

QUIZ #9: CALCULUS 1A (Stankova)
Wednesday, March 31, 2004
Section 11:00–12:00 (Voight)

Problem 1. *Sketch the curve. Find an equation of the slant asymptote.*

$$y = \frac{2x^2 - x - 1}{x+1}.$$

SOLUTION. We first compute the slant asymptote: it has slope $m = 2$, the ratio of the leading coefficients. We compute the intercept:

$$\begin{aligned} b &= \lim_{x\to\infty} \left(\frac{2x^2 - x - 1}{x+1} - 2x \right) \\ &= \lim_{x\to\infty} \frac{2x^2 - x - 1 - 2x^2 - 2x}{x+1} \lim_{x\to\infty} \frac{-3x-1}{x+1} = -3. \end{aligned}$$

Therefore the slant asymptote has equation $y = 2x - 3$.

There is a potential vertical asymptote at $x = -1$, and in fact,

$$\lim_{x\to -1^+} \frac{2x^2 - x - 1}{x+1} = +\infty, \qquad \lim_{x\to -1^-} \frac{2x^2 - x - 1}{x+1} = -\infty.$$

Next, we find critical points:

$$\begin{aligned} y' &= \frac{(x+1)(4x-1) - (2x^2 - x - 1)}{(x+1)^2} = \frac{4x^2 + 3x - 1 - 2x^2 + x + 1}{(x+1)^2} \\ &= \frac{2x^2 + 4x}{(x+1)^2} = \frac{2x(x+2)}{(x+1)^2}. \end{aligned}$$

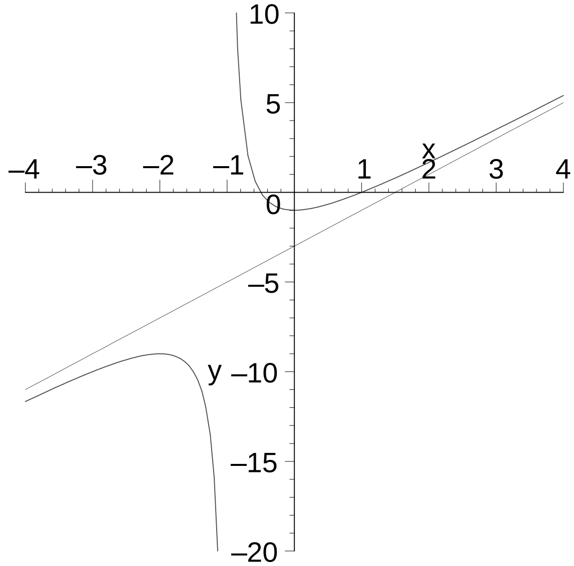
So the critical points are $x = 0$ and $x = 2$. ($x = -1$ is not a critical point, since it is not in the domain of the original function.)

To see if these are local minima or maxima, we can use the First Derivative Test, but it also works to use the Second Derivative Test:

$$\begin{aligned} y'' &= \frac{(x+1)^2(4x+4) - (2x^2+4x)2(x+1)}{(x+1)^4} \\ &= \frac{(x+1)((x+1)(4x+4) - 2(2x^2+4x))}{(x+1)^4} \\ &= \frac{4x^2 + 8x + 4 - 4x^2 - 8x}{(x+1)^3} = \frac{4}{(x+1)^3}. \end{aligned}$$

We see that at $x = 0$, $y'' > 0$ so it is a local minimum, and at $x = -2$, $y'' < 0$ is a local maximum. (You can also compute concavity.)

This gives the following graph:


10
5
–4
–3
–2
–1
0
1
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x
–5
y
–10
–15
–20