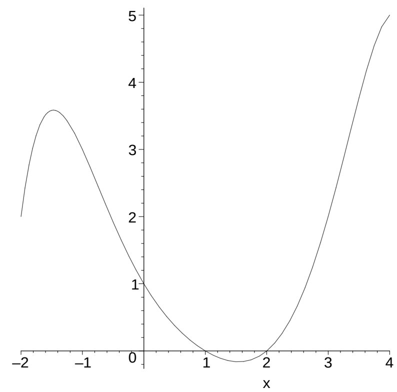
QUIZ #13: CALCULUS 1A (Stankova)
Wednesday, April 28, 2004
Section 10:00–11:00 (Voight)

Problem 1. *Let* $g(x) = \int_{-2}^{x} f(t)\,dt$, *where* f *is the function shown.*

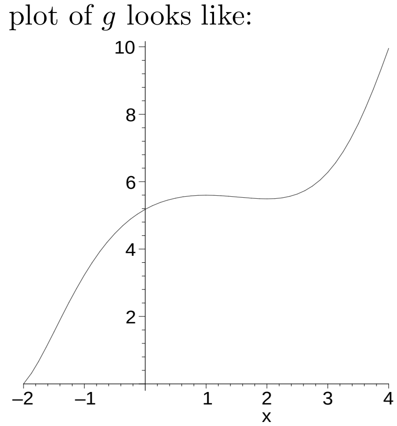


(a) *Evaluate* $g(-2)$.
(b) *Is* $g(4) > 0$*? Explain.*
(c) *Estimate* $g(0)$.
(d) *Where does* g *have a maximum value in the interval* $[-2, 4]$*?*
(e) *Draw a (very) rough graph of* g.

Solution. For (a), we have $g(-2) = 0$ (an integral with endpoints equal is zero). For (b), yes, $g(4) > 0$ because the total area above the axis is greater than that below. For (c), we estimate using right endpoints and the intervals $[-2, -1]$ and $[-1, 0]$ (with $\Delta x = 1$), which has $f(-1) = 3$ and $f(0) = 1$ so

$$g(0) \approx \sum_{i=1}^{2} f(x_i)\,\Delta x = (3 + 1)(1) = 4.$$

For (d), g has a maximum value at $x = 4$, since this is where the total area will be maximal. The plot of g looks like:



QUIZ #13: CALCULUS 1A (Stankova)
Wednesday, April 28, 2004
Section 11:00–12:00 (Voight)

Problem 1. *Use the Fundamental Theorem of Calculus (Part 1) to find the* derivative *of the function*

$$y = \int_1^{x^2} (\sqrt{t} + \ln t)\,dt.$$

SOLUTION. If

$$y = \int_1^{h(x)} f(t)\,dt$$

then

$$y' = f(h(x))h'(x).$$

So since $f(t) = \sqrt{t} + \ln t$ and $h(x) = x^2$, we have

$$y' = (\sqrt{x^2} + \ln(x^2))(2x) = 2x|x| + 4x\ln x.$$

Problem 2. *Use the Fundamental Theorem of Calculus (Part 2) to evaluate the integral.*

$$\int_1^2 \frac{6+\sqrt{u}}{u^2}\,du.$$

SOLUTION. The antiderivative of

$$f(u) = \frac{6+\sqrt{u}}{u^2} = 6u^{-2} + u^{-3/2}$$

is

$$F(u) = -6u^{-1} - 2u^{-1/2} = -\frac{6}{u} - 2\frac{1}{\sqrt{u}}.$$

By the FTC we have

$$\int_1^2 \frac{6+\sqrt{u}}{u^2}\,du = F(2) - F(1) = -3 - \frac{2}{\sqrt{2}} - (-6-2) = 5 - \sqrt{2}.$$