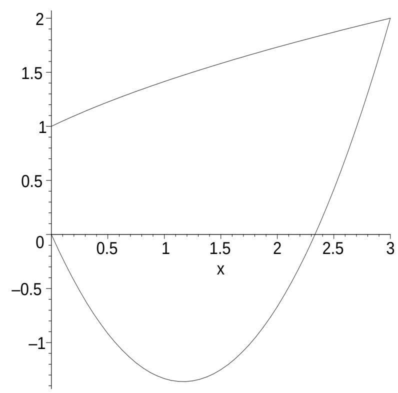
QUIZ #14: CALCULUS 1A (Stankova)
Wednesday, May 5, 2004
Section 10:00–11:00 (Voight)

Name:

Please complete the following problem(s) in the space provided. You may *not* use a calculator. You will have 15 minutes to complete the quiz.

Please include all relevant intermediate calculations and explain your work when appropriate.

Problem 1. *Find the area of the region bounded by the curves* $y = \sqrt{x+1}$ *and* $y = x^2 - 7x/3$ *between* $x = 0$ *and* $x = 3$.



Solution. The area is given by the integral

$$\begin{aligned}\int_0^3 \left(\sqrt{x+1} - (x^2 - 7x/3)\right)\,dx &= \int_0^3 \Big((x+1)^{1/2} - x^2 + 7x/3)\Big)\,dx \\ &= \left(\frac{2}{3}(x+1)^{3/2} - \frac{x^3}{3} + \frac{7x^2}{6}\right)_0^3 \\ &= \frac{2}{3}4^{3/2} - 9 + \frac{63}{6} - \frac{2}{3} = \frac{16}{3} - 9 + \frac{21}{2} = \frac{37}{6}.\end{aligned}$$

QUIZ #14: CALCULUS 1A (Stankova)
Wednesday, May 5, 2004
Section 11:00–12:00 (Voight)

Problem 1. *Evaluate the* indefinite *integral*

$$\int 5x\sqrt{x^2-1}\,dx.$$

SOLUTION. Let $u = x^2 - 1$. Then $du = 2x\,dx$, so

$$\begin{aligned}\int 5x\sqrt{x^2-1}\,dx &= \frac{5}{2}\int (2x)\sqrt{x^2-1}\,dx = \frac{5}{2}\int \sqrt{u}\,du \\ &= \frac{5}{2}\cdot\frac{2}{3}u^{3/2} + C = \frac{5}{3}(x^2-1)^{3/2} + C.\end{aligned}$$

Problem 2. *Evaluate the* definite *integral.*

$$\int_0^1 \left(\frac{1}{2\sqrt{x}}\right) e^{\sqrt{x}}\,dx.$$

SOLUTION. Let $u = \sqrt{x}$. Then $du = (1/2\sqrt{x})\,dx$. For $x = 1, 4$ we have $u = 1, 2$, so

$$\int_1^4 \left(\frac{1}{2\sqrt{x}}\right) e^{\sqrt{x}}\,dx = \int_1^2 e^u\,du = e^u\big|_1^2 = e^2 - e.$$