# MATH 1A: REVIEW OF TRIGONOMETRIC FUNCTIONS

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**Exercise.** *How many degrees is the same as $\pi$ radians?*

*Solution.* Since $360° = 2\pi$, we see that $180° = \pi$.

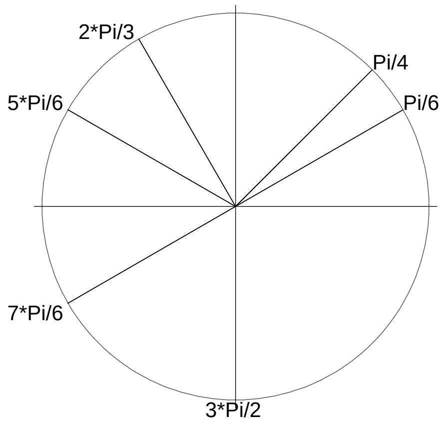
**Exercise.** *Is it true that* $\cos 30° = \cos 30$*?*

*Solution.* *No!* Really not!

**Exercise.** *Fill in the following table. Draw and label these angles on the unit circle.*

*Solution.*

| degrees | $0°$ | $30°$ | $45°$ | $120°$ | $270°$ | $210°$ | $150°$ |
|---|---|---|---|---|---|---|---|
| radians | $0$ | $\pi/6$ | $\pi/4$ | $2\pi/3$ | $3\pi/2$ | $7\pi/6$ | $5\pi/6$ |




**Exercise.** *Recall that* $\pi = 3.14159\ldots$*. Approximately how many degrees is* $1$ *radian?*

*Solution.* Therefore 1 radian is just shy of $1/3$ of a half-arc, so it is just slightly less than $60°$.

**Exercise.** *Label each quadrant indicating whether the functions* $\cos x$ *and* $\sin x$ *are positive or negative.*

*Solution.*

| Quadrant | $\cos x$ | $\sin x$ |
|---|---|---|
| I | $+$ | $+$ |
| II | $-$ | $+$ |
| III | $-$ | $-$ |
| IV | $+$ | $-$ |

**Exercise.** *Fill in the following table.*

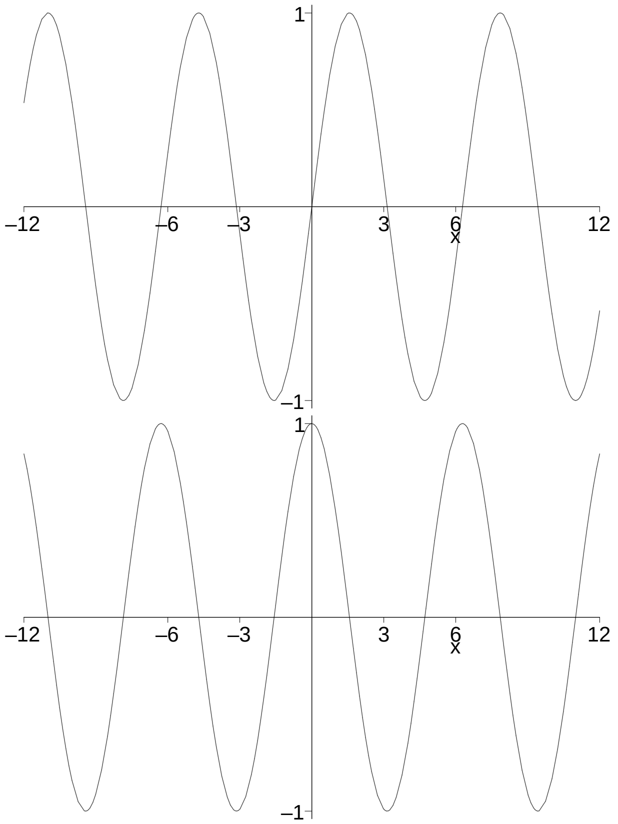
*Date*: February 4, 2004.

*Solution.*

| $x$ | $0$ | $\pi/3$ | $\pi/4$ | $\pi/6$ | $\pi/2$ | $\pi$ | $3\pi/2$ | $-\pi/4$ | $17\pi/6$ |
|---|---|---|---|---|---|---|---|---|---|
| $\cos x$ | $1$ | $1/2$ | $\sqrt{2}/2$ | $\sqrt{3}/2$ | $0$ | $-1$ | $0$ | $\sqrt{2}/2$ | $-\sqrt{3}/2$ |
| $\sin x$ | $0$ | $\sqrt{3}/2$ | $\sqrt{2}/2$ | $1/2$ | $1$ | $0$ | $-1$ | $-\sqrt{2}/2$ | $1/2$ |

**Exercise.** *Draw the graphs of* $\sin x$ *and* $\cos x$*. Use the values tabulated above, if necessary. What are the periods of these graphs? Where does this come from?*

*Solution.*




These graphs have period $2\pi = 6.2...$, which comes from the fact that after $2\pi$ radians we have "gone a full circle", so the values of sin and cos repeat.

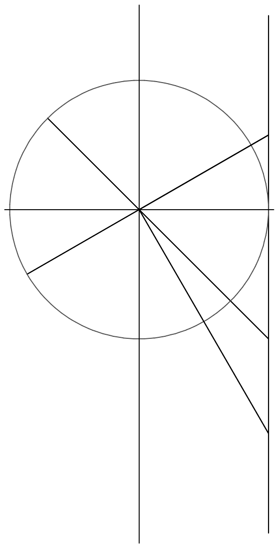
The reason that these functions are entirely contained in the horizontal strip $[-1, 1]$ is because they are the $x$ and $y$ coordinates along the unit circle.

**Exercise.** *For what values of* $x$ *is* $\tan x$ *undefined? For what values of* $x$ *is* $\cot x$ *undefined? [Hint: Check when the denominator is zero.]*

*Solution.* The function $\tan x$ is undefined whenever $\cos x = 0$, i.e. when $x = \dots, -\pi/2, \pi/2, 3\pi/2, 5\pi/2, \dots$. The function $\cot x$ is undefined whenever $\sin x = 0$, i.e. when $x = \dots, -\pi, 0, \pi, 2\pi, \dots$.

**Exercise.** *Draw and compute the value of* $\tan x$ *using this procedure for* $x = \pi/6, 3\pi/4, 7\pi/6, -\pi/3$.

*Solution.*



We can almost read from the graph that $\tan(\pi/6) = \cos(\pi/6)/\sin(\pi/6) = \sqrt{3}/3$, and similarly $\tan(3\pi/4) = -1$, $\tan(7\pi/6) = \sqrt{3}/3$, and $\tan(-\pi/3) = \sqrt{3}$.

**Exercise.** *What happens to* $\tan x$ *when the angle approaches* $\pi/2$ *from below? (What happens to the point of intersection as the angle is just below* $\pi/2$ *and gets closer and closer?)*

*What happens when the angle approaches* $-\pi/2$ *from above?*

*Solution.* Drawing the picture, we see that as the angle $x$ approaches $\pi/2$ from below, the intersection point along the vertical line tends higher and higher upward: this says that

$$\lim_{x\to\pi/2^-} \tan(x) = +\infty.$$

Similarly, approaching $-\pi/2$ from above, we obtain $\lim_{x\to-\pi/2^+} \tan(x) = -\infty$.

**Exercise.** *Label each quadrant indicating whether the functions* $\tan x$ *and* $\cot x$ *are positive or negative.*

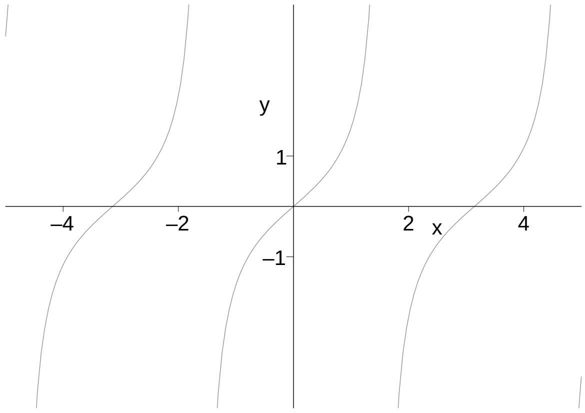
*Solution.*

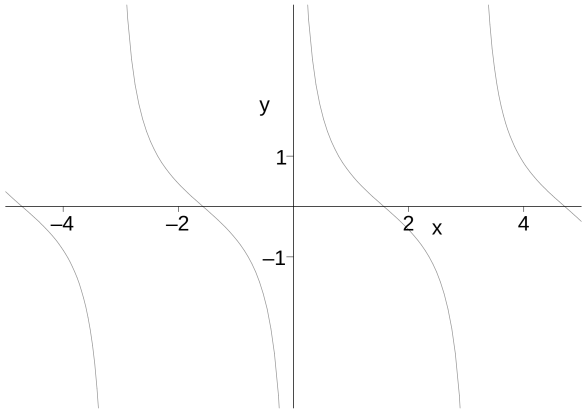
| Quadrant | $\tan x$ | $\cot x$ |
|---|---|---|
| I | $+$ | $+$ |
| II | $-$ | $-$ |
| III | $+$ | $+$ |
| IV | $-$ | $-$ |

**Exercise.** *Draw the graphs of* $\tan x$ *and* $\cot x$*. Recall the values for which these functions are undefined and how they behave near these values.*

*What are the vertical asymptotes of these graphs? Notice that the functions are* not *contained in the horizontal strip* $[-1, 1]$*. Why is this?*

*Solution.*




The vertical asymptotes for $\tan(x)$ occur where this function is undefined, $x = \ldots, -\pi/2, \pi/2, \ldots$, and similarly for $\cot(x)$ at $x = \ldots, -\pi, 0, \pi, \ldots$.

The functions are no longer contained in the vertical strip because we are no longer taking coordinates along the unit circle, but rather their ratios, which can become arbitrarily positive and negative.