MATH 1A: REVIEW OF TRIGONOMETRIC FUNCTIONS

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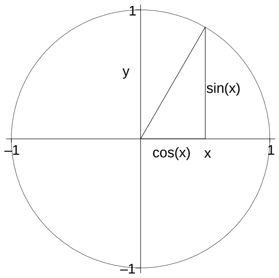
For more practice on the material in this section, please see Appendix D in your text.

Consider the unit circle. The meaning of the functions $\cos x$ and $\sin x$ are the "horizontal" and "vertical" displacement of the second arm with an angle of measure x in radians.



Remember that the angle increases in the counterclockwise direction, and we measure these angles in *radians*, not degrees. The angle given by a complete revolution is 360°, which is the same as 2π radians.

Exercise. *How many degrees is the same as* π *radians?*

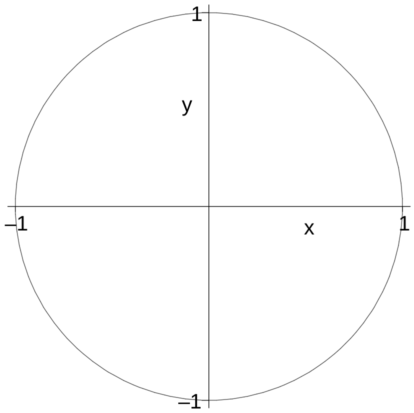
Calculators usually work in radians and problems (if nothing is specified) are meant to be in radians. A lot of errors can be avoided by *not* plugging in degrees for x by mistake!

Exercise. *Is it true that* $\cos 30° = \cos 30$*?*

Exercise. *Fill in the following table:*

degrees	0°	30°	45°				150°
radians	0			$2\pi/3$	$3\pi/2$	$7\pi/6$	

Draw and label these angles on the unit circle:

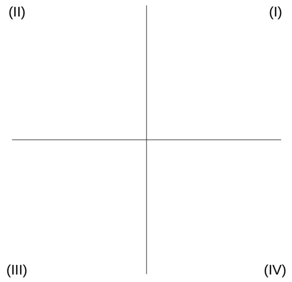


Date: February 4, 2004.

Exercise. *Recall that* $\pi = 3.14159\ldots$. *Approximately how many degrees is* 1 *radian?*

Exercise. *Label each quadrant indicating whether the functions* $\cos x$ *and* $\sin x$ *are positive or negative.*

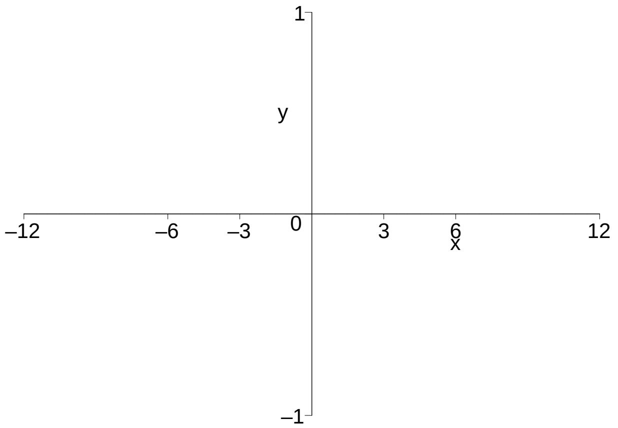
Quadrant	$\cos x$	$\sin x$
I	$+$	$+$
II		
III		
IV		

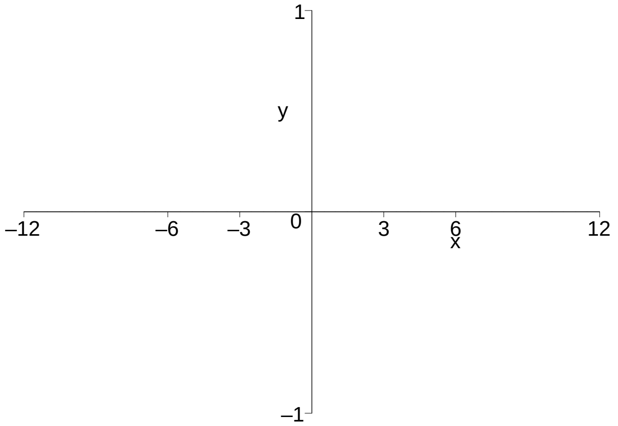


Exercise. *Fill in the following table:*

x	0	$\pi/3$	$\pi/4$	$\pi/6$	$\pi/2$	π	$3\pi/2$	$-\pi/4$	$17\pi/6$
$\cos x$	1								
$\sin x$	0							$-\sqrt{2}/2$	

Exercise. *Draw the graphs of* $\sin x$ *and* $\cos x$. *Use the values tabulated above, if necessary.*





What are the periods of these graphs? Where does this come from?

Notice that the functions are entirely contained in the horizontal strip $[-1, 1]$. *Why is this?*

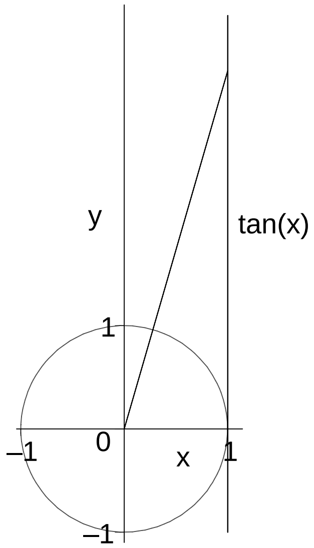
We define

$$\tan x = \frac{\sin x}{\cos x} \qquad \text{and} \qquad \cot x = \frac{\cos x}{\sin x}.$$

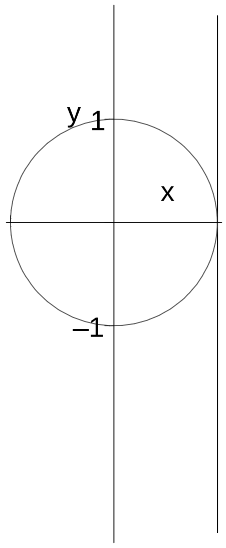
Notice that $\tan x = 1/\cot x$, so these two functions are reciprocal functions.

Exercise. *For what values of x is $\tan x$ undefined? For what values of x is $\cot x$ undefined? [Hint: Check when the denominator is zero.]*

We can interpret $\tan x$ on the unit circle. The function $\tan x$ can be measured by using the vertical line through $(1, 0)$: extend the second arm of the angle until it hits the line at a point; the height of this point is the value of $\tan x$.



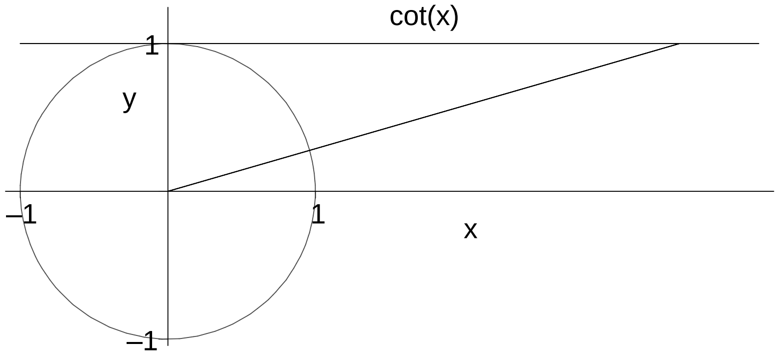
Exercise. *Draw and compute the value of $\tan x$ using this procedure for $x = \pi/6, 3\pi/4, 7\pi/6, -\pi/3$.*



Exercise. *What happens to* $\tan x$ *when the angle approaches* $\pi/2$ *from below? (What happens to the point of intersection as the angle is just below* $\pi/2$ *and gets closer and closer?)*

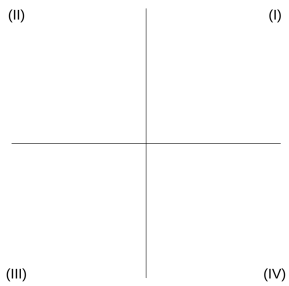
What happens when the angle approaches $-\pi/2$ *from above?*

In a similar fashion, we can interpret $\cot x$ on the unit circle, measured by using the horizontal line through $(0, 1)$: extend the second arm of the angle until it hits the line at a point, and the displacement of this point is the value of $\cot x$.

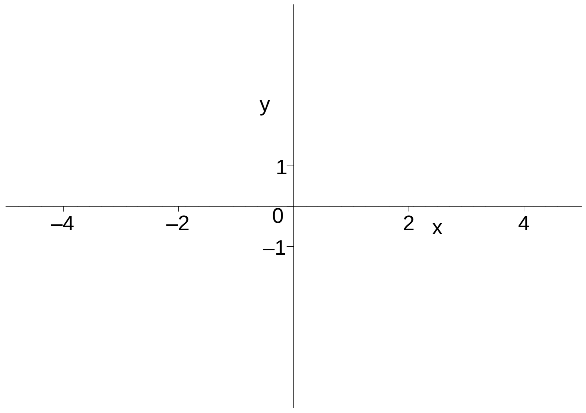


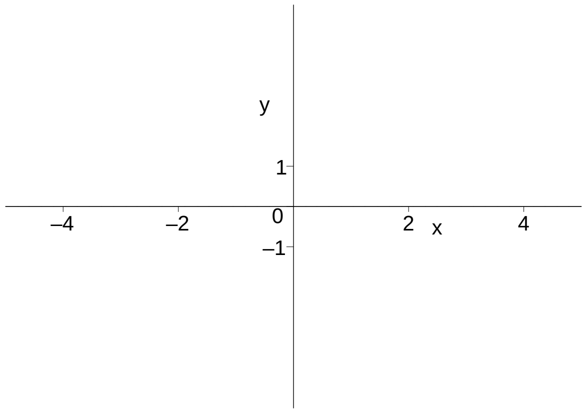
Exercise. *Label each quadrant indicating whether the functions* $\tan x$ *and* $\cot x$ *are positive or negative.*

Quadrant	$\tan x$	$\cot x$
I	$+$	$+$
II		
III		
IV		



Exercise. *Draw the graphs of* $\tan x$ *and* $\cot x$*. Recall the values for which these functions are undefined and how they behave near these values.*





What are the vertical asymptotes of these graphs?

Notice that the functions are not *contained in the horizontal strip* $[-1, 1]$. *Why is this?*