

MATH 252: ABSTRACT ALGEBRA II
HOMEWORK #5A

Problem 1 (DF 13.1.1). Show that $p(x) = x^3 + 9x + 6$ is irreducible in $\mathbb{Q}[x]$. Let θ be a root of $p(x)$. Find the inverse of $1 + \theta$ in $\mathbb{Q}(\theta)$.

Problem 2 (DF 13.1.5). Let $f(x) \in \mathbb{Z}[x]$ be a monic polynomial. Suppose that $f(\alpha) = 0$ for some $\alpha \in \mathbb{Q}$. Show that $\alpha \in \mathbb{Z}$.

Problem 3 (DF 13.2.3–4).

- (a) Determine the minimal polynomial over \mathbb{Q} for the element $1 + i$.
- (b) Determine the degree over \mathbb{Q} of $2 + \sqrt{3}$ and of $1 + \sqrt[3]{2} + \sqrt[3]{4}$.