MATH 252: ABSTRACT ALGEBRA II
HOMEWORK #5B

Problem 4 (DF 13.2.7). Prove that $\mathbb{Q}(\sqrt{2}+\sqrt{3}) = \mathbb{Q}(\sqrt{2},\sqrt{3})$. *[Hint: Consider* $(\sqrt{2}+\sqrt{3})^2$*.]* Conclude that $[\mathbb{Q}(\sqrt{2}+\sqrt{3}):\mathbb{Q}] = 4$. Find an irreducible polynomial over $\mathbb{Q}$ satisfied by $\sqrt{2}+\sqrt{3}$.

Problem 5 (DF 13.2.14). Prove that if $[F(\alpha):F]$ is odd then $F(\alpha) = F(\alpha^2)$.

Problem 6 (DF 13.2.21). Let $D \in \mathbb{Z}$ be squarefree and let $K = \mathbb{Q}(\sqrt{D})$. Let $\alpha = a + b\sqrt{D} \in K$.

(a) Show that the "multiplication by α" map

$$\begin{aligned} \phi : K &\to K \\ \beta &\mapsto \phi(\beta) = \alpha\beta \end{aligned}$$

is a linear transformation (of vector spaces over $\mathbb{Q}$).

(b) Compute the matrix of ϕ on the basis $1, \sqrt{D}$ of K.

Problem 7 (sorta DF 13.3.5).

(a) Show that $\alpha = 2\cos(2\pi/5)$ satisfies the equation $x^2 + x - 1 = 0$. *[Hint: Use a trigonometric identity or the fact that* $\alpha = \zeta_5 + 1/\zeta_5$ *where* $\zeta_5 = \exp(2\pi i/5) = \cos(2\pi/5) + i\sin(2\pi/5)$ *is a primitive fifth root of unity.]*

(b) Conclude that the regular 5-gon is constructible by straightedge and compass.

Date: 24 March 2008; due 4 April 2008.