MATH 252: ABSTRACT ALGEBRA II
HOMEWORK #6

Problem 1 (DF 14.1.5). Determine the group $\mathrm{Aut}(\mathbb{Q}(\sqrt[4]{2})/\mathbb{Q})$ explicitly.

Problem 2 (DF 14.2.3). Determine the Galois group of $(x^2-2)(x^2-3)(x^2-5)$. Determine *all* the subfields of the splitting field of this polynomial.

Problem 3 (DF 14.2.4–5). Let p be prime.

(a) Prove that $K = \mathbb{Q}(\zeta_p, \sqrt[p]{2})$ is the splitting field of $x^p - 2$.
(b) Show that $\mathrm{Gal}(K/\mathbb{Q})$ is isomorphic to the group of matrices

$$\left\{ \begin{pmatrix} a & b \\ 0 & 1 \end{pmatrix} : a, b \in \mathbb{F}_p,\ a \neq 0 \right\} \subset GL_2(\mathbb{F}_p).$$

Problem 4 (DF 14.2.14). Let $K = \mathbb{Q}\left(\sqrt{2+\sqrt{2}}\right)$.

(a) Show that $[K : \mathbb{Q}] = 4$.
(b) Show that K is Galois and $\mathrm{Gal}(K/\mathbb{Q}) \cong \mathbb{Z}/4\mathbb{Z}$.

Problem 5. Let $K/\mathbb{Q}$ be a finite extension.

(a) Prove that $\#\mathrm{Hom}(K, \mathbb{C}) = [K : \mathbb{Q}]$. *[Hint: Use induction on $[K : \mathbb{Q}]$, and follow the proof from class.]*
(b) Let L/K be a finite extension. Prove that every embedding $K \hookrightarrow \mathbb{C}$ extends to $[L : K]$ embeddings of $L \hookrightarrow \mathbb{C}$. *[Hint: Modify the proof as in (a).]*

Date: 31 March 2008; due 18 April 2008.