ALGORITHMS FOR LINEAR ALGEBRA

JOHN VOIGHT

1. Getting Started in Sage, Part II

To get started with the Sage notebook, do one of the following.

(1) Open a web browser anywhere in the world. Go to

`http://www.sagenb.com/`

to open the `sage.math` notebook, hosted by William Stein at the University of Washington. Follow the steps to create an account, and after following the directions in the confirmation e-mail, log in.

(2) Open a web browser anywhere *on campus*. Go to

`https://antigone.uvm.edu:8000/`

to open the notebook, hosted by me! Follow the steps to create an account, and after following the directions in the confirmation e-mail, log in.

(3) Go to Perkins 102 and log in as "sage". The password was just given to you.

Choose Sage from the start menu. A terminal-like window will open. Log in as `notebook`.

```
sage login: notebook
Linux sage 2.6.17-12-386 #2 Sun Sep 23 22:54:19 UTC 2007 i686

The programs included with the Ubuntu system are free software;
the exact distribution terms for each program are described in the
individual files in /usr/share/doc/*/copyright.

Ubuntu comes with ABSOLUTELY NO WARRANTY, to the extent permitted by
applicable law.


------------------------------------------------------------------------


Open Firefox to the address http://192.168.235.128
It may take up to about 10 seconds for the server to start.  Press refresh.
(Click here and press control-C twice to stop the notebook server)


------------------------------------------------------------------------
```

Open Firefox with the above address IP address (yours may vary).

(4) Acquire a computer running Linux, Windows, or MacOS X. Open a browser to the page

`http://www.sagemath.org/download.html`

and download the binary for Sage corresponding to your platform. Install it on your machine and run according to the `README`.

Date: 19 March 2008.

2. Today

To get started today, follow instructions (2). Click on *Published* in the upper right-hand corner. Choose *252 Lab 2: Algorithms for Linear Al....* Then click *Edit a copy.* You're good to go!

3. Labwork

Open a new worksheet and work on the following problems. Print out whatever you complete by the end of class.

Problem 1. Compute the kernel $\ker A = \{v \in V : Av = 0\}$ for the matrix

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 4 \\ -2 & -4 & 0 & 0 & 2 \\ 1 & 2 & 0 & 1 & -2 \\ 1 & 2 & 0 & 0 & -1 \end{pmatrix}.$$

Check that your answer is correct explicitly.

Problem 2. Compute the rational canonical form of the matrix

$$\begin{pmatrix} 1 & 2 & -4 & 4 \\ 2 & -1 & 4 & -8 \\ 1 & 0 & 1 & -2 \\ 0 & 1 & -2 & 3 \end{pmatrix}.$$

Check your work by computing its minimal and canonical polynomial. Compute the Jordan canonical form and compare with the rational canonical form.

Problem 3. How many similarity classes of matrices are there in $M_2(\mathbb{F}_3)$? Play around with:

```
k = GF(3)
M2k = MatrixSpace(k,2)
for A in M2k:
    print A.characteristic_polynomial().factor()
```

What patterns do you notice?

Why happens when you put `A.jordan_form()` in the loop? What goes wrong?

Thinking about rational canonical form, conjecture (and prove!) a formula for the number of similarity classes in $M_2(\mathbb{F}_p)$ for p an odd prime.