MATH 255: ELEMENTARY NUMBER THEORY
EXAM #1

Name _____

Please complete the following problems in the space provided. Please include all relevant intermediate calculations and explain your work.

Problem 1.

(a) Compute $\gcd(24, 103)$.

(b) Find integers $x, y \in \mathbb{Z}$ such that $24x + 103y = 1$ and x is divisible by 5.

Problem 2. For any integer $a \in \mathbb{Z}$, prove that $\gcd(3a + 5, a + 2) = 1$.

Problem 3. Let $m \in \mathbb{Z}_{>0}$ be a positive integer. Show (by induction) that for all $n \in \mathbb{Z}_{\geq 0}$, we have
$$(1 + m)^n \equiv 1 + mn \pmod{m^2}.$$

Problem 4. Let $n \in \mathbb{Z}_{>4}$. Show that $n \mid (n-1)!$ if and only if n is composite.

Problem 5. Show that $\sqrt{1 + \sqrt{2}}$ is irrational.

Problem 6 (Bonus). A random integer n is chosen between 1 and 10000, inclusive. Approximate the probability that n is odd and composite. *[Hint:* $\log(10) \approx 2.5.$*]*