MATH 255: ELEMENTARY NUMBER THEORY
HOMEWORK #7

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7.1: The Euler Phi-Function

Problem 7.1.1. Determine whether each of the following arithmetic functions is completely multiplicative. Prove your answers.

(a) $f(n) = 0$.
(b) $f(n) = 2$.
(c) $f(n) = n/2$.
(d) $f(n) = \log n$.
(e) $f(n) = n^2$.

Problem 7.1.3. Show that $\phi(5186) = \phi(5187) = \phi(5188)$.

Problem 7.1.5. Find all positive integers n such that $\phi(n) = 6$. Be sure to prove that you have found all solutions.

Problem 7.1.11. For which positive integers n does $\phi(3n) = 3\phi(n)$?

Problem 7.1.14. For which positive integers n does $\phi(n) \mid n$?

Problem 7.1.33*. Prove that $\phi(n) = n\prod_{p|n}(1 - 1/p)$ using the principle of inclusion-exclusion.

Problem 7.1.46. Show that if f and g are multiplicative functions, then fg is also multiplicative, where $(fg)(n) = f(n)g(n)$.

7.2: The Sum and Number of Divisors

Problem 7.2.1. Find the sum of the positive integer divisors of each of the following integers.

(a) 35
(b) 196
(c) 1000

Problem 7.2.4. For which positive integers n is the sum of divisors of n odd?

Problem 7.2.5(a). Find all positive integers n with $\sigma(n) = 12$.

Date: Due Wednesday, 18 March 2009.

Let $\sigma_k(n)$ denote the sum of the kth powers of the divisors of n, so that $\sigma_k(n) = \sum_{d|n} d^k$. Note that $\sigma_1(n) = \sigma(n)$.

Problem 7.2.20. Find $\sigma_3(4)$, $\sigma_3(6)$, and $\sigma_3(12)$.

Problem 7.2.21. Give a formula for $\sigma_k(p)$ where p is prime.

Problem 7.2.25*. Find all positive integers n such that $\phi(n) + \sigma(n) = 2n$.