MATH 255: ELEMENTARY NUMBER THEORY
HOMEWORK #8

JOHN VOIGHT

7.3: Perfect Numbers and Mersenne Primes

Problem 7.3.3–4. Find a factor of each of the following integers.

(a) $2^{15} - 1$
(b) $2^{1001} - 1$
(c) $2^{289} - 1$

If n is a positive integer, we say that n is *deficient* if $\sigma(n) < 2n$, and we say that n is *abundant* if $\sigma(n) > 2n$. Every integer is either deficient, perfect, or abundant.

Problem 7.3.5. Find the six smallest abundant positive integers.

Problem 7.3.6. Find the smallest odd abundant positive integer.

Problem 7.3.7. Show that every prime power is deficient.

Problem 7.3.37. Find all positive integers n such that the product of all divisors of n other than n is exactly n^2. (These integers are the multiplicative analogues of perfect numbers.)

Problem 7.3.39*. Show that if n is a poisitve integer greater than 1, then the Mersenne number $M_n = 2^n - 1$ cannot be the power of a positive integer.

7.4: Möbius Inversion

Problem 7.4.1. Find the following values of the Möbius function.

(a) $\mu(12)$
(c) $\mu(30)$
(g) $\mu(10!)$

Problem 7.4.A. Evaluate the sum $\displaystyle\sum_{n=1}^{\infty} \mu(n!)$.

Problem 7.4.10. Show that if n is a positive integer, then $\mu(n)\mu(n+1)\mu(n+2)\mu(n+3) = 0$.

Problem 7.4.13. For how many consecutive integers can the Möbius function $\mu(n)$ take a nonzero value?

Problem 7.4.17. Suppose that f is a multiplicative function with $f(1) = 1$. Show that

$$\sum_{d|n} \mu(d)f(d) = \prod_{p|n}(1 - f(p)).$$

Date: Due Wednesday, 25 March 2009.