

**MATH 255: ELEMENTARY NUMBER THEORY  
WORKSHEET, DAY #17**

**Problem 1.** Show that if  $n$  is odd and  $3 \nmid n$  then  $n^2 \equiv 1 \pmod{24}$ .

**Problem 2.** Show that the product of three consecutive integers is divisible by 504 if the middle one is a cube.

**Problem 3.** Show that if  $p$  is prime, then  $\binom{2p}{p} \equiv 2 \pmod{p}$ .

**Problem 4.** Show that if  $n$  is a positive integer with  $n \geq 2$ , then  $n$  does not divide  $2^n - 1$ .