MATH 252: ABSTRACT ALGEBRA II
HOMEWORK #8

Problem 1 (DF 12.2.3–4).
   (a) Prove that two 2×2 matrices over F which are not scalar matrices are similar if and only if they have the same characteristic polynomial.
   (b) Prove that two 3×3 matrices are similar if and only if they have the same characteristic and minimal polynomials. Give an explicit counterexample to this assertion for 4×4 matrices.

Problem 2 (DF 12.2.10). Find all similarity classes of 6×6 matrices over $\mathbb{Q}$ with minimal polynomial $(x+2)^2(x-1)$. [It suffices to give all lists of invariant factors and write out some of their corresponding matrices.]

Problem 3 (DF 12.3.2). Prove that if $\lambda_1, \ldots, \lambda_n$ are the eigenvalues of the $n \times n$ matrix A then $\lambda_1^k, \ldots, \lambda_n^k$ are the eigenvalues of A^k for any $k \geq 0$.

Problem 4 (DF 12.3.17). Prove that any matrix A is similar to its transpose A^t.

Problem 5 (DF 12.3.22). Prove that any matrix A with entries in $\mathbb{C}$ which satisfies $A^3 = A$ can be diagonalized. Is the same statement true over any field F?