# MATH 252: ABSTRACT ALGEBRA II
# HOMEWORK #10

**Problem 1 (sorta DF 13.3.5).**

(a) Show that $\alpha = 2\cos(2\pi/5)$ satisfies the equation $x^2 + x - 1 = 0$. *[Hint: Use a trigonometric identity or the fact that $\alpha = \zeta_5 + 1/\zeta_5$ where $\zeta_5 = \exp(2\pi i/5) = \cos(2\pi/5) + i\sin(2\pi/5)$ is a primitive fifth root of unity.]*

(b) Conclude that the regular 5-gon is constructible by straightedge and compass.

**Problem 2 (DF 14.1.5).** Determine the group $\mathrm{Aut}(\mathbb{Q}(\sqrt[4]{2})/\mathbb{Q})$ explicitly.

**Problem 3 (DF 14.1.7).** Show that $\mathrm{Aut}(\mathbb{R}/\mathbb{Q}) = \{\mathrm{id}\}$. *[Hint: Since $\sigma \in \mathrm{Aut}(\mathbb{R}/\mathbb{Q})$ restricts to the identity on $\mathbb{Q}$, argue that $\sigma$ is continuous and therefore the identity on all of $\mathbb{R}$.]*

**Problem 4 (DF 13.4.3–4).** Determine the splitting field and its degree over $\mathbb{Q}$ for $x^4+x^2+1$ and $x^6 - 4$.

**Problem 5 (DF 13.5.5).** For any prime $p$ and any nonzero $a \in \mathbb{F}_p$ prove that $x^p - x + a$ is irreducible and separable over $\mathbb{F}_p$. *[Hint: Prove first that if $\alpha$ is a root then $\alpha + 1$ is also a root.]*

*Date*: 6 April 2012; due Friday, 13 April 2012.